MATHEMATICS AS RHYME

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Mathematics is the rhyme of the universe. Such is the second part of my two-part thesis about the place of mathematics in our understanding of God's world. I have already argued in my last paper in favor of a certain framework for understanding the nature of science. Namely, I have argued that it is fruitful to consider that the universe is God's poem. That framework will serve also as the larger framework for my reflections on mathematics.

Within that framework, I now declare that mathematics is the rhyme of the poem. What, then, do I want to suggest by this analogy? Several things. (1) Mathematics has to do with a particular subpart or aspect of the total "poem" of the universe. It cuts across and intersects many other analogies and metaphors within the poem. (2) Mathematics as rhyme is, in a sense, the most "primitive" analogy in the poem; it is based on the simple idea of identity and difference. (3) The possibilities of mathematics-rhyme are deeply bound up with the nature of the "language system" as a whole. The properties are given largely "<u>a priori</u>" by the system, unlike other analogies in the poem which are included in the poem at the discretion of the creator. (4) Mathematics as rhyme functions <u>in the service of</u> the poem as a whole. It enhances the main points of the poem, but it is not ultimately intelligible simply in itself. It is far from having a totally independent purpose. Let me now consider these points in greater detail.

1. Mathematics as an aspect of the "poem"

First of all, then, mathematics is a particular subpart or aspect of the total "poem" which is the universe. I can therefore apply to mathematics some of my general statements about the universe given in the earlier paper. Mathematics is (a) <u>personally</u> structured, (b) linguistically structured, (c) shot through with metaphor and analogy, (d) utterly dependent on God, (e) characterized by <u>development</u>, (f) surprising in its victory over chaos.

Almost everything that I said about science in my first talk can be applied and worked out in the area of mathematics. I am not going to proceed straight-forwardly to do this working out. But doing so would not be trivial. Philosophies of mathematics have often vigorously denied that mathematics was personal, or dependent on God, or at all characterized by development. Mathematics, people feel, is somehow unique among the sciences. Perhaps, they say, it is better not to classify it as a science at all. Mathematics is "independent of the world." Perhaps the discoveries in physics, chemistry, and biology are a "surprising victory over chaos," because we could imagine it to be otherwise. But mathematics is not surprising because it could not be otherwise.

Well, mathematics is indeed different from the sciences. I have tried to capture some of this intuitive feeling for the "independence" of mathematics by characterizing mathematics as "rhyme." With this characterization I point to the distinction between mathematics and other sciences. The other sciences are various kinds of analogies and allegories within the total poem. Mathematics is rhyme acting in coherence with these various analogies.

2. Mathematics as a distinct science

I claim, then, that mathematics is a distinct science. It interlocks with all other sciences, much as rhyme interlocks with and reinforces the other aspects of a poem. But mathematics is not reducible to some other science (like psychology), any more than rhyme is reducible to some other aspect of the poem. Conversely, other sciences are not reducible to mathematics. Physics talks about energies, bodies, and motions in the world, and proposes equations which might have been otherwise. It is not reducible to mathematics, since the equations of mathematics are true in any "world."

I have already talked about the fallacies and illusions of such attempted "reductionisms" elsewhere (Poythress 1976a: 48-54). And before me Dooyeweerd (1969) and others in the cosmonomic school of philosophy engaged in rather extensive explorations and critiques of reductionisms. Hence I will not tarry over this point. It suffices for me to affirm two complementary truths. First, viewing the matter more positively, we can say this. Reductionisms are plausib1e, attractive, even useful and fruitful, because of the stimulus they give to exploring and exploiting manifold analogies within the total "poem." Virtually anything, including mathematics, physics, chemistry, or some sub-discipline within these, can be used as a personal "perspective" for integrating the whole poem. The truths of the sub-discipline, by personal choice or preference, serve as a focal point around which to gather by means of analogy all the rest of the poem.

Second, we can view the matter negatively. Reductionisms oversimplify. They wipe out and smash the richness of meaning in the poem, by a monomania for seeing only one meaning. Why? The existence of two irreducible aspects of the poem in harmonious interaction is evidence of a design and a Designer. Because men would rather flee from God and hide that evidence from themselves, they proclaim that an impersonal explanation "reducing" one to the other is sufficient explanation.

3. Mathematics as a priori truth

My next two points about the nature of mathematics belong together. First, mathematics is the most "primitive" type of analogy in the poem. And, second, its structure is bound up with the nature of the "language system" of the poem. By these aphorisms or analogies I attempt to indicate both the unique subject matter of mathematics and the unique impression that its truths are "a priori."

Let us prepare the ground a little by reflecting on rhyme in the literal sense. The possibility of rhyme and the characteristics of rhyme in poetry are bound up with structures of similarity and difference in a language system. Consider two words like "love" and "dove." They rhyme if (1) the final vowel and any subsequent consonants are exactly the same in the two words; (2) the remaining parts of the two words are not identical in sound. By this definition "sight" and "site" are not "rhyming" words but words identical in sound (homonyms). Thus the phenomenon of rhyme derives from properties of both identity and difference in the phonemic system or sound system of the language. The potential for rhyme is "primitive" in the sense that it is based on very elementary properties of the phonemic system. The phonemic system in turn is the simplest and most basic of the language systems.

Now let us compare this with mathematics. Mathematics likewise has to do with properties of identity and difference in the universe - in God's macro-poem. It focuses on the very most "elementary" properties, the properties of identity and difference, in the universe. This focus on identity and difference determines its unique perspective or subject matter. Simultaneously, that focus helps to explain the apparently a priori character of mathematical truth. Again let us return to poetry. The possibilities for different rhyming syllables, for masculine rhymes, feminine rhymes, imperfect rhymes, and the like, are given "a priori" by the language system, before a poet sets his pen to paper. The monolingual can hardly conceive of rhyme being other than what it is in his system. Similarly, in mathematics we are all, in a sense, monolinguals. We have experience of only one universe. It is difficult to conceive of an alternative mathematics, because our thoughts are thoughts within a single created "system". Mathematics is a statement of the fixed properties of the "rhyming" possibilities of that system.

From what I have just said, you might suppose that I am making all of mathematics a matter of contingent rather than necessary truth. I appear to be saying that our inability to imagine things otherwise is a limitation in our created mind and in the creation around us, but not a limitation from God's point of view. Is that so? Not necessarily. I am saying that we are finite. Our view of possibility must not legislate for what might be possible for God under vastly different conditions. But I would also like to affirm that God always acts consistently with his own nature. It is not true that God can do anything at all. He cannot lie, he cannot deny himself, he cannot change, and so on. God does whatever he wishes (Ps. 115:3). His wishes are always consistent with who he is.

Now, this has implications for mathematics. Mathematical regularities are a reflection of the faithfulness of God. Thus it may be that a large portion at least of ordinary low-level mathematics would necessarily hold in any universe that God might create. Let me again use the analogy of language. There is indeed more than one possible human language; there is more than one language system. But all human language systems have some structural properties in common. Within certain bounds, all are capable of rhyme. All human languages are characterized by certain constraints because of the nature of humanity. Analogously, we might say that all "systems of possibility" within which God speaks (creates) a universe-poem are constrained by the nature of God.

In addition to this, there is also at least some degree of a posteriori character in our knowledge of mathematics (cf. Poythress 1976a: 168-172, 1974: 134-138). A poet's particular selection of rhymes is still open to him within the limits of a particular language. Likewise, even given a "system," God's choice of what particular things will be identical and different, in what particular ways, is open to him.

4. The subject-matter of mathematics.

I have already given a preliminary indication of the subject matter of mathematics by saying that it has to do with the properties of identity and difference in God's poem. But this is an oversimplification. More is involved in mathematics than simply properties of identity and difference. How are we to set the boundaries as to what is mathematics? What is the difference between mathematics and logic? Between mathematics and mathematical physics? Are statistics and game theory properly part of mathematics? How are we to answer such questions?

Well, it seems to me that such questions about boundaries partly - but only partly – boil down to "semantic" questions. There is more than one way of drawing a boundary. The analogy between mathematics and rhyme may once again illustrate. Rhyme in the narrowest sense is closely related to a number of other regularities of pattern in poetry. One thinks of imperfect rhymes (e.g., between "pure" and "fewer," assonance and alliteration, poetic meters, extended patterns of rhyme (e.g., the sonnet), onomatopoeia, homonymy. We are confronted here with a number of phenomena which can either be included under a single large umbrella term, or carefully distinguished from one another. The fineness of the distinctions depends on the perspective and taste of the observer. Likewise, "mathematics" may be considered as a larger or smaller area of investigation.

That is, "mathematics" as a term may be used to cover a larger or smaller area. I think that I come somewhere near the ordinary scope of the word "mathematics" when I say that mathematics has to do with three or four interlocking areas of investigation, together with the relations between these areas and their ramifications. These areas are (a) properties of identity and difference, (b) properties of quantities, (c) properties of space, and (d) properties of motion. The study of these areas leads to corresponding academic disciplines: (a) elementary set theory, concerning the properties of aggregates ("agorology"), (b) number theory and elementary algebra, (c) geometry, and (d) kinematics (see Poythress 1976a:179-180). Area (d), kinematics, concerned with properties of motion, I include only optionally. Kinematics is usually not considered to be part of mathematics. But I judge that the limit concept in calculus depends ultimately on intuitions about motion. Hence it seems to me that there is much in the field of mathematical analysis that interacts directly with a somewhat redefined conception of kinematics.

At any rate, all will agree that mathematics has now advanced to an impressive depth and complexity, partly by studying higher-level regularities involved in agorology, number theory, and geometry; partly by studying the regularities in the interactions and interconnections between the three or four fields. It is not my purpose, then, to offer a detailed or definitive classification of higher reaches of mathematics. My intent is to suggest some of the sources for mathematics. Mathematics finds its sources in various types of intuition about primitive properties of the universe: identity, quantity, space, motion.[1]

This bare-bones account of the nature of the subject matter of mathematic needs to be filled out in two directions. I need first to say something about the relation of mathematics to some kindred disciplines that I seem to have left out: logic, linguistics, and psychology. I will then say something more about further subdivisions within mathematics, and the possibilities of "reducing" one subdivision to another.

5. Classical reductionistic explanations of mathematics: logicism, formalism, intuitionism, and empiricism

How is mathematics related to logic, to linguistics, and to psychology? Some philosophers of mathematics have gone so far as to claim that mathematics is actually a subdivision of logic (logicism), or of linguistics (formalism), or of psychology (intuitionism). I, on the contrary, have argued above that mathematics has a subject matter of its own distinct from any of these fields. If I am to justify that claim more thoroughly, I should give some account of the plausibility of these competing claims.

5.1 Global basis for plausibility of reductionisms.

To give such an account in general terms is not too difficult. Mathematics forms one aspect of the universe-as-poem. From this I have already inferred that mathematics is personally structured and linguistically structured. Since mathematics is linguistically structured, it should be no surprise that formalism in the philosophy of mathematics has tried to reduce mathematics to language pure and simple: "mathematics is the study of formal languages."

Likewise, mathematics is personally structured. For intelligibility, there must be a personal interpreter. Hence, it is not surprising that intuitionism in the philosophy of mathematics has tried to reduce mathematics to a branch of psychology: "mathematics is the study of <u>mental</u> mathematical constructions."

To explain the basis of logicism is not quite so easy. We could start with the motif of God as a <u>person</u> who is self-consistent in all that he does. Or we could start with the motif of language as a self-consistent organized system. Or we could start with the motif of victory over chaos. God's poem is not a chaos. Because of this, we can make inferences and predictions from observations about some aspects of the poem, and have them vindicated by other aspects. Order, regularity, and the possibility of inference pervade the poem. Hence mathematics also is subject to inference. In fact, mathematics as the study of very "primitive" properties of the "poem" is easier to subject to detailed inferential patterns than are academic disciplines whose subject matter is less primitive. Hence the plausibility of say, "Mathematics is a branch of logic."

So far I have said nothing about the fourth of the "classical" positions in philosophy of mathematics, namely empiricism. Empiricism says that mathematics is a generalization from experience of the physical world. My root metaphor of mathematics as rhyme accounts for this almost automatically. As rhyme occurs in a poem, so mathematics "occurs" or rather "holds true in particular cases" in the world. Is my own position, then, simply a variation on empiricism? Almost, but not quite. Remember that I argued that mathematics, at least from a human point of view, has largely an <u>a priori</u> character because of its interest in the "language system" behind any possible piece of the poem. The old empiricism did not account for this <u>a priori</u> element. Nor did it account for the compatibility between the intuitions of the human mind and the empirical facts "out there."

5.2 The usefulness and "success" of reductionisms

The attractiveness of reductionisms can be understood even better using the idea of multiple perspectives developed in my earlier paper. According to this idea, the same subject matter can frequently be explained or systematized using more than one point of view. More than one root metaphor, more than one "model," can sometimes be developed. In the course of development, there is a kind of reciprocal interaction between the principal subject (the thing modeled) and the subsidiary subject (the model used). The structure, of the subsidiary subject stimulates the investigator to try to extend and deepen the model in certain directions. Contrariwise, the structure of the principal subject causes modifications, tinkerings, closer definitions, and ad hoc additions to the model. The modifications of the model enable it to survive when unpalatable evidence shows up. (For a detailed account of this process, see Kuhn (1970).

The four classical philosophies of mathematics can themselves be considered as instances of this type of development. For all four of them, the works of mathematicians are the subject matter, the principal subject. But the four use different root metaphors (cf. Pepper 1970) or subsidiary subjects as models for making intelligible this principal subject. For logicism, the subsidiary subject is logic. For formalism, it is language. For intuitionism, it is the human mind and its psychology. For empiricism, it is certain physical aspects of the nonhuman world. The "success" of the four philosophies simply demonstrates the fruitfulness of considering mathematics from each of the four viewpoints or perspectives. It demonstrates, in other words, the fruitfulness of a certain analogy or correspondence.

In the process, there is a mutual enrichment. On the one hand, the principal subject, mathematics, is better understood as people try to re-express it in logical terms, in formalist terms, etc. On the other hand, there is also modification of the subsidiary subject. Logic, language, psychology, and physics are each "enlarged" beyond their former boundaries in the attempt to encompass mathematics. For instance, logicism and formalism must each include specifically mathematical axioms in their foundations (such as the axiom of infinity and the axiom of reducibility in the Whitehead-Russell system). And the reader must know how to interpret or apply certain theorems in a mathematical sense, if he is to profit from them.

Intuitionism and empiricism have difficulties of a somewhat different kind. In common forms of intuitionism and empiricism, a great deal of classical mathematics must be abandoned or modified because it is non-intuitive or non-empirical. Alternatively, the concepts of mathematical "intuition" and of the "empirical" that can be boldly and imaginatively expanded to encompass the full range of what mathematicians do. But then, after this radical expansion, is anything worth-while left of the original attempt at reduction?

5.3 The failure of reductionisms to deal with multiple perspectives

Reductionist philosophies of mathematics, then, are stimulating as metaphors, but inadequate as ultimate explanations. I do not intend to review here the criticisms of reductionist philosophies already put forth by rival reductionisms (cf. Benacerraf-Putnam 1964) or by antireductionist philosophies (Dooyeweerd 1969, Vollenhoven 1918 1936, Strauss 1970 1971 1978, Poythress 1974 1976a 1976b). Beyond these criticisms, two more points need to be made.

First, philosophy of mathematics needs to account not only for mathematics but for the plurality of plausible philosophies of mathematics! I would argue that nothing short of a multi-perspective approach to mathematics will succeed here. As in linguistics (cf. Pike 1967:68-72, 84-92, 1980), so in mathematics, only a multi-perspective approach is needed to do justice to both the subjective and objective poles at work in the subject area. On the subjective side, the subject's choice of a perspective, a root metaphor, or a paradigm as a starting point for systematizing his understanding is decisive for the final form his theory will take. On the objective side, the fact that the universe as God's poem includes many built-in metaphors forms the basis for successful development of more than one explanatory model.

Second, using certain insights from Paul Benacerraf (1965), we can show simultaneously the fruitfulness of multi-perspective thinking and the failure of reductionisms. I have in mind an article by Benacerraf entitled “What Numbers Could Not Be.” What is the point of Benacerraf's article? In brief, Benacerraf argues that we know for certain that numbers are not sets. The argument rests in the end on a simple but far-reaching observation. There is more than one way of using sets to “stand for” or designate numbers. In standard Zermelo-Frankel set theory, the nonnegative integers are given by

$$0 = \emptyset$$

$$n + 1 = n \cup \{n\}.$$

In order, they are $\emptyset, \{\emptyset,\{\emptyset\}\}, \{\emptyset, \{\emptyset\},\{\emptyset, \{\emptyset\}\}\},$

$\{\emptyset,\{\emptyset\},\{\emptyset,\{\emptyset\}\},\{\emptyset,\{\emptyset\},\{\emptyset, \emptyset\}\}\}\}$

In an alternate version, they are given by

$$0 = \emptyset$$

$$n + 1 = \{n\}$$

viz. $\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\ \{\{\{\emptyset\}\}\}, \ldots$

Either of these versions, or still others, is an acceptable basis for number theory. Now if numbers were sets, pure and simple, at least one of the above two accounts of numbers would be self-contradictory. In fact that neither is contradictory shows that numbers are not sets. Rather, there exists, on the basis of a set theory, the possibility of establishing a stipulatory correlation between numbers and certain infinite recursive progressions of sets. There is a correlation (an analogy) rather than a metaphysical identity. The fact that there is more than one way to establish a correlation shows that it is a correlation and not an identity.

Benacerraf's argument is thus an antireductionist argument ("numbers are not sets") based on the use of multi-perspectives (multiple possible correlations between sets and numbers). Benacerraf also uses multiple perspectives more positively. By examining which correlations between numbers and sets "do the job," he helps to determine what is "essential" to number.

Now let us apply a similar technique to the four reductionist philosophies of mathematics. Logicism can be seen to be inadequate, because there is more than one way of embedding mathematics in logic. Numbers can be represented in more than one way by sets, as we have seen. And sets can be represented in more than one way in logical formalism.

Similarly, formalism fails for much the same reason. If formalism tries to include a theory of the relations between formal theories, it can do so only by a regress of meta-languages.

Intuitionism is not so easily criticized in this fashion. The genius of intuitionism is, in fact, to insist that numbers (and perhaps space) are sui generis. But multiple perspectives still challenge intuitionism more indirectly. Can intuitionism account for the existence of multiple correlations between (say) the number-system-as-intuited and recursive sequences of sets obeying the Peano axioms? Can it deal with the multiplicity of different people's senses of mathematical "intuition," ranging from extreme finitists to formalists who temporarily adopt formalized intuitionist logic?

Empiricism is also subject to criticism using multiple correlations. Straightforward empiricism in mathematics establishes a correlation between numbers and collections of objects. "Four" is a kind of generalization from experiences of collections of four apples, four fingers, etc. But one can establish correlations in a different way. "Four" can be applied to collections of abstractions ("second-order collections") as well as collections of "things" ("first-order collections"). {2, 5, 7, 8} is a collection of four numbers; {red, green, blue, brown} is a collection of four colors. Can empiricism account for such a generalization cutting across "types"? There is another problem. "Four" can apply to collections that can be divided in more than one way. Four pairs of shoes are also eight shoes; four limbs are one body. Numbers are not "given" in the world in any simple way. They require the subjective contribution of a personal interpreter making decisions as to what differences and identities are relevant to his interests. Four pairs of shoes can be either an instance of four or an instance of eight, depending on the perspective.

6. Multiple correlations in the subparts of mathematics

Using Benacerraf's principle of multiple correlations, we can also construct arguments for showing the nonreducibi1ity of various subparts of mathematics to one another. To provide a first set of examples, let us focus on the four sub-areas of mathematics already distinguished. Mathematics deals with (a) identity and difference, (b) quantity, (c) space, and (d) motion. Can we show that these four are not reducible to one another?

Benacerraf's original argument already shows that numbers cannot be equated with sets. Hence (b) is not reducible to (a). Second, space is not reducible to set theory, since more than one set-theoretic formulation can represent the same geometry. Space is not reducible to number, since there is more than one way of coordinatizing a space. What about the reduction of motion to space or quantity? The same motion can be represented quantitatively in more than one way, depending on the choice of time coordinate. We need to choose both the point of origin for the coordinate and the scale of measurement. Moreover, in order to represent motion in purely spatial terms quantitative time must be transformed into another spatial dimension. Again this can be done in more than one way.

This pattern of argument is in fact capable of demonstrating still further irreducibilities. Ordered pairs are not reducible to sets, since more than one stipulative definition will work. Nor are functions reducible to sets of ordered pairs. Groups are not reducible to an ordered triple consisting of a set, a binary operation of multiplication, and a unary operation of inverse. For groups can also be defined starting from a set with a single binary operation of multiplication (inverse being defined only later in the group axioms). Or groups can be defined using the single binary operation $f(a,b) \equiv a \bullet b^{-1}$ instead of the binary operation $g(a,b) \equiv a \bullet b$.

7. Radical irreducibility

If one approaches matters in this way in every area of mathematics, one is well on the way to a radical extension of the idea of irreducibility. Up to now, I have applied the idea of irreducibility only to broad areas of study. Quantity, space, and motion represent such broad areas. But irreducibility can also be used in narrower cases. From my point of view, nothing is "identical to" or "reducible to" anything else. To take a most outrageous example: the number 12 is not "reducible to" 11 + 1. (It could be defined as 11 + 1, but also as 10 + 2, 9 + 3, 2 x 6, etc. Hence none of these is the correct definition from the point of view of logical deduction.)

To be sure, in many cases stipulative definitions are capable of serving as a starting point for deducing all the important properties of the entity so defined (the definiendum). For example, the stipulative definition 12 = 11 + 1 can be the starting point, in the context of the Peano axioms, for deducing the properties of 12. But that only shows that there is a detailed analogy, not an identity, between definiendum (e.g., 12) and definiens (structures used to do the defining, e.g. 11 + 1 and Peano axioms). The definiens and the definiendum are serving respectively as the subsidiary subject and the principal subject of a mathematical "allegory." Stipulatory definitions are the starting points for so many allegories.

I do not say that this is the only way of looking at mathematical definition. But it is useful for several purposes. I will now focus on two of these purposes.

7.1 Awakening wonder

First, I intend by this "allegorical" approach to reawaken our awareness of wonder in mathematics. We know that it is useful to consider functions as ordered pairs, or to coordinatize Euclidean space. This is something to be wondered at. Even the deducibility of properties of 12 from 12 = 11 + 1 is ultimately mysterious (cf. Wittgenstein 1967:13-16). It is something to praise God for. It is not simply a bare identity calling for no reaction, or "So what?" Our response can be wonder, whether or not the truths in question are a priori or a posteriori from one or another point of view. For in either case they are rooted in the wisdom of God.

Consider by contrast the effect of the pronouncement that "of course it works." The person says, "Of course," because "functions are nothing but ordered pairs in the first place," or "coordinatizability is merely the inevitable consequence of Euclidean axioms," or "12 is nothing but an alternate name for 11 + 1." Even if these statements were truer than they are, they would be an evasion of the ultimately personal character of creation originating in a creator. To repeat what I have said before: let us consider mathematical truth not as simply unproblematically "there," but as a victory over chaos, in fact a constantly reasserted victory.

7.2 Awakening creativity

My second purpose in using an "allegorical" approach is to stir creativity. Once the spell of "ordinariness" is broken, we can let our imaginations play and find alternate "allegories." When we allow ourselves to imagine what it would be like for the original allegory to break down, we are freed to produce creative alternatives. We may find, for example, non-Euclidean geometries, fuzzy functions (cf. Zadeh 1956, Kandel 1974), alternate number systems.

Independent of my own thinking on creativity, William J. Gordon (1961) has developed a theory of creativity emphasizing personal involvement, empathy, fantasy, and emotions as useful aids in technological invention and business. He is much more specific about techniques of creativity than I can be here. But we have both emphasized the involvement of the person of the investigator in knowledge. I can illustrate how this works by taking as an example the positive integers. How can there be creativity here, since the facts are (apparently) so cut and dried? Well, there is, of course, creatively involved in the discovery of new proofs in number theory. But I want to exercise creativity on a far more basic level.

To do so, I personify the integers. I visualize not an infinite series of bare symbols 1,2,3,4, ..., but a row of people. The successor relation I visualize by having each person lay his hand on the shoulder of the next one, or by having each person throw a ball to the next one. Then I fantasize about the ways in which the number system could break down or behave differently. What could happen? All sorts of things. The people could form themselves into a circle instead of a straight line. We would have modular arithmetic. Or at certain points the line could split in two, and we would have a discrete partial ordering. I could imagine each person juggling many balls instead of just one which he passes to the next. Then we have the beginning of the concept of order pairs. I could imagine running out of persons to continue the line, so that the last person had to keep his ball. This corresponds to the finite universe that Whitehead and Russell had to eliminate with their axiom of infinity.

8. Mathematical meaning as meaning in relationship.

Finally, my “allegorical” approach or “poetic” approach to mathematics also encourages a useful emphasis on the relational aspect of mathematical truth and the mathematical understanding. What do I mean by relational aspect? I wish to argue that to understand and appreciate a truth of mathematics is to understand it in relation to many other truths both inside and outside the area of mathematics. (Cf. earlier claims to this effect in Poythress 1976b:172-173.)

In poetry, rhyme finds its significance, its effectiveness, its raison d'etre, not purely in itself, but in its functions in the larger whole. Likewise mathematical truth finds its significance not merely in itself, but in relation to applications and parallels in other areas of mathematics, plus applications in physics, economics, and still other areas. Of course, I want to affirm vigorously that the attempt to “purify” mathematics, to isolate general principles from the specific practical contexts in which they first appeared, has been quite fruitful. But the preference for pure abstraction over concrete embodiment is both one-sided and ineffectual, from a pedagogical as well as a philosophical point of view. Teachers know very well that group theory is best learned when worked-out examples of particular groups are sprinkled in with theorems. Calculus is best learned when examples with particular functions accompany its theorems.

Moreover, the best tests of mathematical knowledge come through applications. For instance, a student who can quote the theorems, explain their meaning, and even repeat the proofs still does not really “know” calculus or group theory unless he can work problems. I would suggest that it is best to treat this pedagogical fact as a fact constitutive for the nature of mathematical truth. It is not simply an inconvenient limitation, a falling short of the Platonic ideal, a concession to the limited powers of men of dust. Remember that Plato was against the body and its “messy” corruption of the pure vision of the abstract ideal. Plato was against creation, in fact. But a Christian ought not to be. The pedagogical constraints are not “unfortunate” corruptions, but an aspect of the created structure of mathematical knowledge.

Pedagogically, then, I am in favor of the reintroduction of the writhing dirty masses of applications into mathematical explanation. One can still keep the abstract generalizations with their Apollonian beauty. But the particular examples are not to be “reduced” to the generality. We ought to revive our wonder for the fact that the generality actually holds for this case, and for that case, and for this other case. Each discovery of a new application can be seen as a development of mathematical truth, the writing of a new line to the poem.

Notes

1. If one is willing to apply a good deal of imagination, one can work out the analogy between mathematics and poetic rhyme even to include this detail. Properties of identity and difference in mathematics correspond to the identity and difference necessary for true rhyme. Properties of quantity correspond to meter in poetry, with its quasi-quantitative count of feet. Properties of space correspond to the structural patterns of regular rhyming schemes (e.g., the sonnet).

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