MAKING CURRICULUM DECISIONS
AND
THE NATURE OF MATHEMATICS

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This morning we will be devoting our attention to pedagogical questions. The written materials I have handed out were cribbed from a paper that I gave in a "Principles to Practice" conference sponsored by Christian Schools International. (Christian Schools International is an agency that serves independent Christian Schools throughout the U.S. and Canada.) But I have also used them as a basis for class discussions on several occasions with Calvin students. For use in the classroom the general plan is as follows:

1. I motivate the students by getting them to think about their own mathematical training and the emphases made in their classes in mathematics. I also relate to them what has happened in mathematics at Calvin in the last two decades.
2. I present the classes with some contrasting opinions about the nature of mathematics and given an analysis of each view.
3. I also outline what realism (Platonism), formalism, logicism, and intuitionism are.
4. I then present some statements that give a <u>pluralist's</u> view of mathematics.
5. I then analyze the present course to see which aspects of mathematics one finds in the course as taught.
6. Perhaps we refer to this material on the final examination.

Let me begin by presenting a description of mathematics at Calvin College in the sixties and seventies. I have deliberately overstated the differences in order to catch your interest. I am sure that you will recognize it as a more or less correct description of what are definite trends at your college or university (with exceptions, of course). After that I will show you the statements of contrasting views and the synopsis of pluralism.

Those of us who have been teaching mathematics during the sixties and. seventies have seen some rather radical changes take place during this time both in our curricula and in our pedagogy; in <u>what</u> we teach and in <u>how</u> we teach. When I first came to Calvin there was great enthusiasm to introduce new courses in mathematics. This was the spirit of the day. There were so many subjects we had learned in graduate school which were not represented in our undergraduate offerings that we were almost impatient in our desire to get them introduced into the curriculum. We had ambitions to teach integrated calculus and analytic geometry in the <u>freshman</u> year. I believe that I taught abstract algebra for the first time at Calvin during my first semester. Thereafter followed a multitude of new courses--point set topology, linear algebra, real analysis, linear geometry, mathematical logic, and complex analysis. For a decade and a half there was lively interest in pure mathematics in our department. Students were unashamedly interested in mathematics for its own sake and many went on to further graduate study in the subject.

Contrast that situation with the status today. Not that there is less overall interest in mathematics than before, but we have become primarily a service department. Furthermore, our upper level courses in pure mathematics have all but dried up. Few students seek to major in our department as pure mathematics majors any longer (much to our chagrin). We teach two tracks of calculus, meeting the needs of a wide range of students including economics--business, biology, and psychology majors. They eagerly learn matrix theory, linear algebra and linear programming, probability theory and statistics. On the upper levels we have two courses in mathematical statistics rather than one; we have thriving courses in numerical analysis, in the mathematics of engineering and physics, and in complex variables. We have designed a formidable and extensive major of concentration in computer science. Some of our students have gone into actuary science and operations research. Obviously, mathematics that is immediately applied has gained the prominence in our curriculum. Mathematics for its own sake is not of interest to our students nor is it regarded as important in our society.

And what has happened in our classrooms? Think of the changes that have taken place in our pedagogy. In the sixties we paid careful attention to the language of mathematics and we presented a hierarchy of precise definitions to our students. Our mathematics was abstract and self-contained. We posed questions arising within the discipline of mathematics; we made conjectures and we gave counterexamples. More importantly, we gave proofs for our theorems and we even required our students to reconstruct them and to formulate their own. In contrast, today we have much less time for the "niceties" of mathematics. Our textbooks are filled with problems that apply the concepts that are introduced. Gigantic efforts are being made to produce motivational materials and settings in which the mathematics we teach gets applied. Rather than emphasizing the language, the symbolism, and the structure of mathematics, we are seeking to build mathematical models to solve problems that arise from societal needs and from our cultural and scientific efforts.

But what is it that determines the content of the mathematics courses we teach and also the style and stance we take *in* the classroom? Are we only passively involved in all of this? Are we merely the victims of outside pressures that are forcing us to reluctantly give up our old ideas of what should be taught in the mathematics classroom? True we are under societal and economic pressure to make the mathematics we teach more useful and relevant to the needs of today’s world. Besides our students are demanding courses that they perceive to the best preparation for today's job market. Furthermore, most of us do not write the textbooks we use and have a profound effect on our style of teaching. Again, we desire to be recognized as respectable and "with it" centers of learning and we eagerly consult with fellow colleagues as to what they are doing.

But is there more to it than this? Have we, in fact, as individuals changed our philosophical stance about the nature of mathematics and its role in the education of our youth? Many of us have enthusiastically embraced the current emphasis and have contributed towards the efforts at change. Are there principial reasons for such change? Shouldn't we develop a philosophy of mathematics that will serve as a touchstone for making the curriculum decisions that face us? It is my opinion that each of us teachers of mathematics should do some hard, principial thinking about the nature of mathematics which will help us decide which new courses we should propose and which teaching techniques are most appropriate for us to use in the mathematics classroom. In this way our mathematical course content and our style will not be merely what mathematicians deem important or what they are doing at the moment.

Here are some areas where contrasting opinions produce tension and disagreement among those that teach mathematics and among those who design curricula and write materials.

The Role of Man

A. Man is the creator of mathematical structures and systems. If there is order in things around us it is because, by mental construction, man has imposed it on things.

B. Man is the discoverer of the mathematical order that exists in the things around us.

Relationship to Other Disciplines

A. Mathematics is an independent, autonomous discipline that has its own questions, its own content, and its own distinctive methodology.
B. The importance of mathematics derives from the fact that it is a functional tool in the solution of problems from everyday life and in the other disciplines.

Methodology

A. Mathematics is distinctive in that it is a deductive science.
B. Deduction is no more important in mathematics than in other disciplines. It is only one way among others that is available to substantiate and undergird mathematical findings. It does not produce mathematical results.

Role of Language

A. Mathematics is also distinctive in its use of concise and symbolic language. Moreover, it is useful in mathematics to give careful definitions of technical terms and to identify the hierarchical structure of the terms that are used.
B. The construction of concise languages is not an end in itself. Language is for communication and there is little point in asking for more precision of language than is appropriate in a given situation.

Nature of the Discipline

A. Mathematics is a unified, coherent discipline. The notions of set, relation, and function are examples of some fundamental concepts that give unity to it. It is a worthwhile enterprise to try to reduce (unify) mathematics by extensive use of such fundamental notions.
B. Mathematics is a heterogeneous discipline that deals with several aspects of reality. Methods that apply in one area may not be appropriate in other areas.

Truth

A. Mathematics does not deal with truth. It deals with abstract structures having similar form.

B. Mathematics is important in that natural laws are formulated in its language.

Purpose of Mathematical Activity

A. Mathematics is to be enjoyed for its own sake. The game of mathematics has aesthetic appeal and there is great enjoyment in its pure beauty.
B. The ultimate purpose of mathematical activity is to better understand and control the world.

Role of Axiomatics

A. The creation of axiomatic systems is an important mathematical activity. In so doing we discern the first principles of a body of knowledge and also logical relationships between its findings.
B. There is little point to axiomatizing a body of knowledge. The results are the important things while logical relationships between them are of little importance.

Now to a more controversial part. What follows is a list of descriptive statements about the discipline of mathematics. It is a pluralistic approach and attempts to put a Christian viewpoint on these matters. It is my opinion that it is unwise to give a reductionistic account of mathematics, but rather to give a balanced account that recognizes the role of the content of mathematics and also that of the person who is creating and/or learning it.

1. Mathematics has its roots in God's creation order, in man's ability to count and measure, and in his capacity to discern shapes and patterns.

2. An important goal of mathematical activity is to better understand natural and social phenomena around us.

3. As a creature of God, created in His image, man is able to distinguish the numerical and spatial properties of things, to represent them pictorially, to name them, to abstract, to imagine never-ending processes, and to generalize. This mental activity results in the consideration of the possible (the way things may be) and in the consideration of infinite classes of abstract objects. Mathematics is rightly called the "science of the infinite."

4. There is a beautiful duality in mathematics in that it concerns the arithmetic and the geometric--the discrete and the continuous. These fundamentally irreducible aspects are nonetheless intimately related. In the solution of mathematical problems we constantly change our perspective from arithmetic to geometric and vice versa. This duality is best exemplified in the numbers.

5. The basic arithmetical laws in the creation order are so pervasive and fundamental and so firmly rooted in things around us that they are quite readily accessible to us through our everyday experiences. Thus there is a finality and certainty about our knowledge of these laws. This is certainly not the case with the geometric. Here we are far less certain of our formulations.

6. Men use physical models, pictorial representations, intuition, inductive methods, analogy, and deduction to discover mathematical laws and theories. Since mathematics is a communal enterprise, these findings must be substantiated and communicated to others. Because of the nature of the objects under consideration (abstract, infinite collections), logical, deductive processes are vital. Thus axiomatic formulation plays an important role in more mature sub-disciplines of mathematics. In formulating theories axiomatically, one better sees and exhibits the foundation stones and the logical relations between the various theories.

7. Another distinctive, human activity that is especially fruitful in mathematics is that of creating language and symbols to carry mathematical meaning. Corresponding to underlying mathematical laws, men devise symbols and rules for their manipulation. Such activity makes *our* calculations and arguments more reliable and more accessible to the scrutiny of others. Such mathematical language becomes universal.

8. Mathematics is an alive, mature, and growing discipline. As it continues to examine its foundations, it is constantly reorganizing and renewing itself. The mathematical community of scholars is heterogeneous in that there is no general agreement on the meaning and purpose of mathematical activity.

9. Mathematics is a human enterprise with a long, illustrious history. It is a chronicle of human efforts to understand. It is replete with examples of how one's view of life, one's faith commitments, determines theoretical activity.

10. A crowning achievement of mathematical activity is to be able to apply its results to the better understanding of the phenomena around us. Yet, since it deals with the possible and the imagined, it has a certain autonomy of its own. It invariably asks internal questions thereby creating mathematical theories which surround the applied theories and which deepen our understanding of reality. Thus, the autonomy of mathematics does not necessarily work to the detriment of the other disciplines, but strengthens and deepens them.

11. Mathematics is a heterogeneous discipline containing such diverse subjects as number theory, analysis, topology, geometry, probability theory, statistics, and combinatorics. Yet, fortunately, there are fundamental notions which give unity and coherence to the discipline and which provide a common setting for its considerations. Among these are set, function, relation, and operation.

There are two reasons why our approach is pluralistic and eclectic. Both are fundamental tenets of the Christian faith. The first is that the God of the Scriptures is the Lord of His creation and that, ultimately, He is the source of all of our knowledge of His universe. On the other hand we have a high view of man. We believe that, despite his sin, man is the bearer of the very image of God. As such he is endowed with very special gifts. He is a responsible, moral creature who is called to be a servant of God, living his entire life in loving response to Him.

All of this means that we disavow a view of mathematics which makes man an independent, autonomous, self-sufficient being who by his mental effort creates mathematical systems thereby imposing the order and design he observes around him. Man is not the lawgiver of the universe. The basis for mathematics lies in the wise and orderly decrees of the revealed God of the Bible. He is the Creator of the order in and about us. In His orderly design and in his lawful decrees, He is showing His faithfulness to His creation. Yet, our view of man as created in God's image leads us to assert a certain amount of creativity and autonomy for him. He discerns bits and pieces of God's law structure and. he makes conjectures about the way God's law operates. Thus he is culturally active in the mathematical process and he is creative in the way he formulates his theorems and in his use of logic and language.

Nor would we recognize any special infallible status for the axiomatic, deductive methods of mathematics as if man can by identifying certain "self-evident" statements and by deduction find absolute truth. Furthermore, within the discipline of mathematics, man uses many techniques besides the purely deductive to discover mathematical laws. Intuition, induction, experimentation, guessing, and reasoning by analogy are among the techniques available. On the other hand, since mathematics arises from our ability to image the possible the never-ending, the infinite, we need deductive reasoning to substantiate our conjectures. It is a God-given gift that men are able to arrange mathematical findings in an orderly way as a series of logically interrelated statements and proofs.

Again, we recognize that although mathematics is important in its relationships to the other disciplines, its ability to make explanation of the very complex phenomena of the physical, economic, psychological, and social is limited. The spatial and arithmetic are only aspects of these phenomena and all attempts to reduce them to the spatial and arithmetic are bound to fail.

Mathematics is no different from the other disciplines in that experiences in learning and teaching mathematics are shaped by one's beliefs. History has many examples of mathematical theories, which are strongly influenced, by religious belief. Classroom teachers should present mathematics in its historical perspective and point out these relationships. So, too, our emphases in our mathematics classrooms are influenced by our theology, our view of man's place in the world, by our belief about the meaning of life, and by our belief about the nature of the child. The classroom teacher should articulate these views in presentations.

There is a need to build to the highly conceptual and abstract parts of mathematics through concrete, real-life experiences with material things. Such informal processes play a vital role in the development of intuition and are the basis for our abstraction and generalizations. This is especially the case in the classroom of the elementary school. Students must initially "read" God's world in order to articulate mathematical results. Nor should this contact be broken at any level of maturity. Again, at all levels the classroom teacher must show the relationships of mathematics to the other disciplines. The circle of mathematics must be completed in that it must be applied to the better understanding of phenomena around us.

Yet, mathematics is not to be identified with physics, chemistry, or economics. Down through the ages there has developed a well-defined area of knowledge of the spatial and arithmetic. Mathematics has a well-developed, universally used symbolic language, which must be carefully taught to the student. It has a time-honored way of communicating and substantiating its findings, namely using deductive reasoning and proof. It has its own agenda of questions, which are to be answered. The world of mathematics is no less real than its applied parts, as it also owes its existence to God. Its beauty and profoundness deepens our appreciation of the power and majesty of God.

All of this means that we are not frightened or turned away by the abstractions and generalizations of mathematics. For there is real beauty and design here too. In fact, history shows us that if our applications of mathematics are to be lasting and profound, then our abstractions and creative imagination must be highly developed and creative.

By way of application, let me list some things that we should continue to do in our classrooms as we go into the eighties.

1. Let us try to give a balanced portrayal of the discipline of mathematics to our students, thereby resisting over-emphasis upon the deductive self-contained side and also upon the applications.

2. Let us continue to teach in the context of applications of mathematics--on both sides--as motivation and as application of ideas presented.

3. Let us continue to train our students in the technique of building mathematical models to solve problems.