A COMPARATIVE STUDY OF CHRISTIAN MATHEMATICAL REALISM AND ITS HUMANISTIC ALTERNATIVES

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I can vividly recall as a boy lying on our living room floor listening to our little radio and hearing the Metropolitan Opera on Saturday afternoon. Even though the words were sung in a tongue foreign to me, I recall being much impressed by the majestic music of the orchestra, the loud vibratos of the soloists and the enthusiastic applause of the audiences. Being brought up in a religious family, I imagined that such opera must be concerned with the sublime and lofty. Only later did I find that some of the subject matter was, at best, mundane and even banal.

One might say that a similar misconception applies to the discipline of mathematics today. Such eminent spokesmen as Morris Kline, in Mathematics: Loss of Certainty[1] and David & Hersh in The Mathematical Experience[2] are informing the educated laymen that their conceptions about mathematics as a collection of indubitable truths are naive, out of date, and badly out of harmony with historical fact.

Much of what has been said is critical of a view of the nature of mathematics known as "mathematical Platonism." Nicholas Goodman has nicely characterized the view in the following way: to the Platonist... "Mathematics consists of truths about abstract structures existing independent of us, of the logical arguments which establish these truths, of the constructions underlying these arguments, of the formal manipulations of symbols that express these arguments, and truths and nothing more."[3] Thus, there is an ontological side to Platonism in that it professes belief in abstract, eternal structures, independent of man, but also an epistemological side in that it asserts that man is equipped with an intuition which provides access to these objects and which guides our explanations and arguments concerning them.

It is interesting to note that many practitioners of mathematics admit to a psychological stance that is most consistent with a p1atonistic view. Davis and Hersh say "most writers on the subject seem to agree that the typical mathematician is a Platonist on weekdays and a formalist on Sundays." Paul Cohen writes that the "… Rea1ist's position is probably the one that most mathematicians would prefer to take. It is not until he becomes aware of some of the difficulties in set theory that he would even begin to question it. If these difficulties particularly upset him, he will rush to the shelter of Formalism, while his normal position will be somewhere between the two, trying to enjoy the best of two wor1ds."[4]

Generally speaking, the current philosophical opinion about mathematical realism is very negative. Could it be that there is even a kind of abhorrence of these ideas? One detects some of this abhorrence and distain in the pejorative language that is used to describe this ethereal realm of mathematical objects -- ranging from mathematical "zoo" to a "fairyland." True, over the years such prominent mathematicians as G.H. Hardy, René Thom, and Kurt Godel have expressed their general agreement with a realist's view. Thom writes, "Everything considered, mathematicians should have the courage of their most profound convictions and affirm that mathematical forms indeed have existence that is independent of the mind considering them... Yet, at any given moment, mathematicians have only an incomplete and fragmentary view of the world of ideas."[5] Godel, in speaking about mathematical intuition says "I don't see any reason why we should have less confidence in mathematical intuition than in sense perception ... These objects, too, may represent an aspect of objective reality."[5]

Now among those of us who are mathematicians and also professing of Christians there naturally arises the following question: What kind of stance do we have regarding these questions? Again, Davis and Hersh comment on the relation between science and religious belief by saying, "It is the writer's impression that most contemporary mathematicians and scientists are agnostics; or if they profess to religious belief they keep their science and their religion in two separate boxes." Implicit in this statement is the opinion that religious belief will have little to say which is enlightening regarding these questions. But, again, Christians profess belief in the existence of the Triune God of the Scriptures. This God is the creator of the cosmos and is providentially upholding His creation, even now. Shouldn't this belief have something definitive to say about the existence of abstract mathematical entities and man's access to them? I say this reverently. We Christians profess belief in God, a concrete but non-material person. Does this fact "break the barrier," so to speak, and make it plausible to believe in the existence of other non-material, yet abstract entities? Davis and Hersh hint at such a possibility for theists when they write, "Belief in a non-material reality removes the paradox from the problem of mathematical existence, whether in the mind of God or in some abstract and less personalized mode. If there are realms of non-material realities, then there is no difficulty in accepting the reality of mathematical objects which are simply one particular kind of non-material object."

Perhaps as you face the questions raised, you also feel some abhorrence, distaste, and distrust. Presumably such abstract entities have necessary existence and are eternal. Wouldn't the existence of such universals such as properties, states of affairs, possible worlds, and numbers present a challenge to God's sovereignty and rule? Doesn't the Bible rule out the possibility that there is anything else besides God the Creator and the created Cosmos? Can it be that there are eternal entities or structures, which are independent of God and not subject to His control?

It is interesting to note that these questions have been treated only recently within the scholarship of Reformed philosophers. Nicholas Wolterstorff in his book entitled "On Universals" argues that the religious significance of the traditional creator-creature distinction does not demand that it be exhaustive of all reality.[7] First of all, it is not clear that when the Biblical writers spoke, they had universals in mind when they spoke of "all things." The Biblical writers were not advocating some abstract, theoretical ontology. They were expressing their conviction that God has a claim on man's promise and obedience and also that man can rely upon God to be faithful to His creation order. Thus, the existence of universals that are neither identical with God nor created by Him is not a threat to the religion of the Biblical writers.

Wolterstorff goes even further. When speaking of the over-arching ontological milieu of "predictable/case/exemplification" which he has identified in his philosophical analysis, Wolterstorff says "Nothing is unique in that it falls outside this fundamental structure of reality. God too has properties; he too acts. So he exemplifies predicables. The predicable/case/ exemplification structure is not just the structure of created things..."

I would be remiss if I gave the impression that the matter is settled among Reformed theologians and philosophers. One need only read the somewhat heated exchange between Michael Morbey and Anthony Tol in the June 1981 issue of the journal Anakainosis[8] about whether there is a disagreement between Dooyeweerd and Vollenhoven regarding the ontic status of the "law" or "law-idea" as a boundary between God and the Cosmos to see that there is much more work to do.

Alvin Plantinga's Views

In his recently published little book called Does God have a Nature? Alvin Plantinga deftly tries to create some logical room for the existence of eternal, abstract entities that are uncreated.[9] His case is for a form of Christian Platonism. It rests on the question of whether God has a nature. I shall merely summarize his argument.

First, he meets the objection commonly raised by fellow Christians who reject the entire question as improperly raised.

> They say: God transcends human experience; we cannot observe or in any other way experience him.

So our concepts do not apply to God and hence questions about whether He has a nature are not for us to ask. Plantinga shows that such an argument is self-referential and cannot be coherently maintained. He counters by saying that this position is "a pious and commendable concern for God's greatness and majesty and augustness, but ends in agnosticism and incoherence."

But what does Plantinga mean by asking, "Does God have a Nature?" By this he means to ask whether God has properties that are essential to Him; properties that he could not fail to have. Technically x has a property P essentially iff x has P in all possible worlds where x exists. Now, if God has such an essential property P, then it isn't up to him whether or not he has that property--his having it is in no way dependent on his own decision or will. For example, if omniscience is a property of God that is essential to Him, then there never was a time when God was not omniscient and hence his having this property was in no way dependent upon his action. In general then, neither God's existence nor his character seems to be in His control. But, of course, this runs counter to another judgment that we often make about God and His creation, namely that nothing exists that was not created by God and that there is nothing besides Him which is not within His sovereign control.

In order to portray some of the flavor of this Plantinga argumentation I shall outline its main points. You will see that the argument creates a tension between saying that God is sovereign and exists "a se" and that he has nature. The formulation is his.

(1) God is sovereign and exist "a se" (uncreated).

(2) God is alive, knowledgeable, capable of actions, powerful and good.

(3) If (1), then (a) God has created everything distinct from Himself (b) Everything distinct from God is dependent upon Him (c) He is not dependent upon anything distinct from Himself (d) Everything distinct from Him is within His control.

(4) If (2), there are properties such as having life, knowledge, etc., and God has these properties.

(5) If God has these properties distinct from Him, he is dependent upon them.

(6) God is a necessary being.

(7) God is essentially alive, knowledgeable, capable of action, and good.

(8) If (7) then there are properties such that God could not have failed not to have

them.

(9) So, such properties could not have failed to exist.

(10) If God has some properties that exist necessarily, and are distinct from Him, then God is dependent on these properties (in some sense), they are independent of Him, uncreated by Him, and outside His control.

Of course, there are several ways to relieve the tension of having to draw consequence (10). The nominalists say that there are no such things as properties. Plantinga counters by saying that Christian nominalists would still say that the state of affairs that God is alive and knowledgeable is still not in God's control and thus a threat to His sovereignty. He also meets the objections of those, who like Aquinas hold that God has no properties distinct from Himself (Divine Simplicity) and also of those like Descartes who hold that God has no <u>essential</u> properties (universal possibilism).

To summarize, then, the argument is that there are some abstract entities (properties or states of affairs), which God has not created and which are outside of His control - benignly so, but outside of His control, nonetheless.

In the last chapter of his book, Plantinga proceeds to treat such abstract, necessarily existing objects in general. The argument goes as follows:

(1) God's having a nature is equivalent to there being some necessary propositions.

(2) Omniscience is a property included in God's nature.

(3) So, since God exists and necessarily so, every proposition p will be equivalent to the proposition that God believes p.

(4) So, for any necessarily existing abstract object O, it is part of <u>God's</u> <u>nature</u> to affirm the existence of O. Also, for any necessary <u>proposition</u> p, it is part of God's nature to believe p.

To be specific, if 7 exists, then God has essentially the property of affirming that 7 exists and conversely. Furthermore, if 7 + 5 = 12 is necessarily true, then it is part of God's nature to believe that 7 + 5 = 12 and conversely.

Plantinga summarizes the discussion to say, "exploring the realm of abstract, necessary objects can be seen as <u>exploring the nature of God</u>." Even though these abstract, necessary objects are not in God's control, it is part of God's nature to affirm each and every truth about them.

Before making an evaluative comment, let me include two definitions that may be helpful. They concern the concepts of "dependence" and "control."

<u>Definition 1</u>: y depends upon x for property P iff there is an action to such that x can perform A and such that x's performing A is a necessary condition for x's having P.

<u>Definition 2</u>: x is in control of y's having property P iff there is an action A such that x can perform it and such that if x performs A, then y does not have property P and also an Action A* such x can perform it and if x performs A* then y has the property P.

Notice that by this definition 7 depends upon God for existence, even though 7's existence is not within God's control.

Plantinga's contribution to the philosophy of mathematics, as I see it, is to use the time-honored concepts of God's nature which we Christians profess, to argue for the existence of other abstract necessarily existing objects like those under scrutiny in man's mathematical efforts. Furthermore, one may regard this lack of control as benign to say the least. These essential properties that God has present no threat to God in any way. They are not agents of action that oppose God.

God's revealing to us that He has these properties enhances and enriches our concepts of Him. The same may be said for numbers, for example. They present no threat to God's sovereignty and again only better contribute to our understanding of His greatness and goodness.

The Views of Vern Poythress

By way of contrast, let us consider another Christian thinker who takes very seriously what we Christians say about the Creator God and attempts to see what this has to do with the ontology and epistemology of mathematical entities. I have in mind the views expressed by Vern Poythress in the chapter entitled A Biblical View of Mathematics that appeared in the book Foundations of Christian Scholarship, edited by Gary North.[10] If you study the Poythress material and compare it with Plantinga's book you will be struck by the difference in style of presentation. Poythress' work is far less analytical, but certainly deals more concretely with the relevant Biblical texts than Plantinga's; it is Biblically informed and deserves careful scrutiny.

At the start Poythress makes a valuable contribution to the discussion by identifying three common uses of the word “mathematics.” He says it may refer to

(a) The historically growing science found in textbooks, articles, conferences, lectures, etc. Here mathematics is considered as the cultural product of human thought which has accumulated over the centuries and which has been tested by the mathematical community and which has been included in the corpus of mathematics as a result.

(b) The thoughts of individual mathematicians. By this he, no doubt, means the mathematical consciousness of individual mathematicians and included in this are the mental constructions that they make.

(c) The mathematical structure for the world, which exists independent of our thoughts.

It is evident from this classificational scheme that Poythress does subscribe to idea that there is mathematical structure and order in the created cosmos and that this law structure exists independent of human thought processes.

Concerning mathematics (c) then, we may ask the very basic question: Are mathematical structures aspects of creation or of God? Poythress' answer follows from the following considerations.

(1) God is sovereign over all. Everything must find its meaning and its very existence in Him.

(2) The most basic ontological distinction in Scripture is between God on the one

hand and His creatures on the other.

(3) Mathematical structures and laws are not created by God. “At no time does the Bible speak about God's having created structures or laws.” Poythress gives such structures or laws no independent status. God action is immanent with His By creation; it is not mediated to the creation by laws independent of the creator.

(4) Mathematical structures and laws are thus of God - they belong to God's nature. The Bible tells us about God's nature, its aggregative, its numerical, its spatial, and its kinematic aspects. These aspects are not only present in God's dealing with his creatures, but they were present in God's nature before the creation and “appear to go beyond the created world into eternity.”

(5) Rather than requiring that God correspond to our ideas derived from this created world, we should rather see, conversely, that our mathematical thoughts (mathematics (b)) are derived from God's imprint and governing rule over finite things.

(6) Since man has been created in God's image he is capable of understanding God, albeit only partially. He has the ability to understand the aggregative, the numerical, the spatial, and the kinematic aspects of God's rule. In exploring mathematics, one is exploring the nature of God's rule over the universe--that is, one is exploring the nature of God, Himself.

The following is a listing of the major points of difference between Plantinga and Poythress.

Plantinga

-The approach is theological, philosophical.

-God's nature is described in terms of necessary properties that He possesses, in static terms. The absolute, necessary nature of these properties requires the judgment that the creator-creature distinction is not exhaustive.

-God is present within a milieu having aspects that are not within God's control yet in which God finds expression and character and in which he can act.

-God's having a nature also requires the existence of necessary propositions. For any propositions p, p is necessarily true iff it is part of God's nature to affirm p. Thus to discover such necessary propositions is to explore God's nature. Since the propositions of mathematics are necessary to discover their truth is to discover something about God's nature.

Poythress

-The approach is biblical and revelational.

-God's nature is described in terms of God's actions, in dynamic terms.

-There is God and the creation and nothing else.

-Mathematical structures and laws reside within God.

-Since the impress of God's actions have been placed upon finite things, there is a reflection of God's nature on the creation order.

-Man, being created in God's image is capable of understanding God, in a creaturely way.

-To see the aggregative, numerical, spatial, and kinematic aspects of God's creation order is to understand God's nature and his action with the cosmos.

Philip Davis and Reuben Hersh's ideas

By comparison the approach taken by Davis and Hersh in their book The Mathematical Experience is completely secular and in a completely different direction. I shall list the main points of their analysis:

(1) We need not retreat into formalism when attacked.

(2) Neither do we have to admit that our belief in the objectivity of mathematical truth is Platonic in the sense of requiring an ideal reality apart from human thought.

(3) The usual categories of supposing the world as containing only two kinds of material, matter (physical substance), and mind (my mind or your mind), are inadequate.

(4) Following Karl Popper, there are 3 worlds or categories available to us. They are the physical world, the world of individual consciousness "which emerges from the material world in the course of biological evolution," and the world of social consciousness, tradition, language, theories, and social institutions.

(5) Mathematics is an objective reality that is neither subjective nor physical. It is an ideal (non-physical reality) that is objective and is external to the consciousness of anyone person.

(6) Mathematics is part of the world of social consciousness. It is part of human studies. It is capable of establishing reproducible results (proofs, diagrams, physical manipulation of things).

(7) Mathematics does have a subject matter and its statements are meaningful. Its meaning is to be found in the shared understanding of human beings. It is similar to an ideology, a religion, an art form. It deals with human meaning and it is intelligible only within the context of culture. It is a humanity.

(8) Mathematicians invent ideal objects and then try to discover facts about them. Thus mathematics is a human invention in that we mentally create ideal objects, but we can discover properties of these objects only by great effort and ingenuity.

(9) What distinguishes mathematics from the other humanities is its science-like quality. Its conclusions are compelling in that they are argued for.

(10) So, mathematics is fallible, connectable, and meaningful.

Regarding mathematical epistemology, Davis and Hersh have some extremely interesting comments to make, especially for teachers of mathematics. They propose to account for the phenomenon of mathematical intuition by examining more carefully the methods that we use to teach mathematics. The essential questions are as follows: What do we teach? How do we teach these things? As they describe it, the process of learning mathematics has these aspects.

(1) We teach mathematical concepts by developing a mathematical intuition within the individual student. This intuition is not a direct perception of something existing externally and eternally.

(2) The individual is triggered to learn by carefully designed experiences. They consist of manipulating physical objects, of drawing pictures, of seeing examples, of doing problems, of constructing arguments. These experiences leave a trace on the individual mind. The individual's response is that a representation of the concept is formed in his mind.

(3) To check whether the individual's representation is the socially sanctioned one, the student is invited to check with others to see whether his representation is "correct." "We compare notes and figure out what is right." As an authority figure (the teacher) presumably has the proper representation and she uses social conditioning (bad marks!) to refine the representations of the individual students.

(4) As a result of this long conditioning process students learn the essential ingredients of mathematics (here regarded as a humanity) and those that become professional mathematicians get to invent new concepts and to answer questions regarding them.

(5) Summarizing then, the real objects of mathematics are these shared concepts (mutually congruent representations) which we men and women have developed over the centuries.

(6) What is unique about mathematics is that these shared concepts are reproducible. Thus, mathematics is the study of mental objects, mutually shared, with reproducible properties. (I take it that "reproducible" means that a properly trained person can reproduce or repeat an experience or thought process as described by another person and thus "verify" the mathematical assertions of the other as part of results accepted by the community of mathematical scholars.)

Before I present my reaction to all this, let me remind the reader of the various uses of the word "mathematics" which Poythress identified. I shall make an addition for purposes of my analysis:

Mathematics is:

A: the historically growing science found in textbooks, articles, journals, libraries

B: the thoughts of individual mathematicians

C: the mathematical structure for the world that exists independent of our thoughts

D: the science of spatial and arithmetical universals that exist independent of our thought.

It is clear that Davis and Hersh (DH) concern themselves exclusively with A and would deny the fruitfulness of dealing with C and D. As it is with so many attempts to describe complex phenomena, what DH say about mathematics has a measure of truth as descriptions of A and B. The characterization that DH give is a good antidote to an overemphasis upon man's autonomous reasoning abilities to produce infallible truth. Certainly some of the mathematical assertions included in Mathematics A and many of which are in B are fallible, corrigible, and subject to error. Any cultural artifacts produced by mankind will be imperfect and fragmentary.

In addition, the description given by DH is a good corrective to the unrealistic objectives behind the gigantic effort to "found" mathematics in the first half of the twentieth century. To see mathematics in its historical context as the product of men in a cultural context with its successes and its foibles is only being faithful to the historical account of how mathematics developed. As teachers we serve our students well by better acquainting them with the saga of mathematics.

Having said this I must also add that I consider DH's approach to be distorted from a Christian perspective. Let me say briefly why I think this is the case.

DH close their book by characterizing mathematics as "fallible," "correctible," and "meaningful." Describing Mathematics A and B as fallible is easy to agree with. The latter two are terms that deal implicitly with content (to be meaningful requires that the sentences of mathematics concern themselves with something) and e with correctness or truth (to be correctible requires that there is a standard or norm which selects some sentences as correct and some as incorrect). By restricting themselves to Mathematics A and B, DH must make their entire case within the confines of these categories. Their approach is of necessity purely secular and also purely humanistic. Furthermore, it is my assertion that such a strategy leads to difficulties and tensions that are as difficult to remove as the difficulties they discern in the Platonistic accounts. This is due to the fact that they are forced to give explanatory status to certain phenomena that do not adequately carry an account of meaning and truth. Let me be more specific.

One thing that DH do in their account is to reify the cultural product of the mathematical community that is recorded in the mathematical literature. It is this product that is the record of the common mathematical understanding as passed down from generation to generation through the centuries. But just what is this cultural product and what is the motivation for recording it? Certainly it is not to be identified with the mathematical sentences that are found in journals and textbooks. Such identification would be a return to formalism. But what is it if it does not refer to something about which it is making assertions? What is there about these sentences to recommend their inclusion in the body of mathematics? Are they merely a record of the psychological gymnastics of the generations? Why preserve such? Raymond Wilder makes the ultimate generalization when he asserts, "it is not only practical but theoretically sound to treat culture as a super-organic entity which evolves according it is own laws." I am assuming that the subculture of mathematics could be treated as such an entity. What is difficult for me to envision is just what kind of laws there would be to govern such evolution. Wouldn't such laws be as difficult to account for as the mathematical "laws" which DH are so skeptical of?

A second device that DH use is to reify the "common understanding" which men have regarding mathematical concepts. Presumably, there is formed within our minds a representation of the mathematical concepts which we invent. We "rub" our representations against those of others so as to form congruent representations that are common and correct. But how is such a process any less mysterious than the mathematical intuition that was the subject of so much derision from DH?

Thirdly, DH reify the process by which mathematicians currently decide which theorems properly belong to the corpus of mathematics and detach these sentences from their content. It seems that the truth of p is to be identified with what some ideal, rational, scientific community allows me to assert at a given time. Again, as a Christian, it is difficult for me to imagine that exclusively cultural forces determine the mechanism that determines the norms and standards by which mathematical sentences are judged without slipping into obscurantism and chaos. More importantly, I find little dynamic within this purely cultural explanation to account for the amazing power of the applications of mathematics to real-world problems.

What I say must not be taken as denying the validity of viewing mathematics as cultural product. As Christians we take man's cultural and scientific efforts seriously--regarding them as having a measure of success in progressively producing better understanding of the cosmos we live in. Yet, the attempts by DH, Kline, Wilder and others to characterize mathematics as a humanity and redefining meaning and truth without reference to an objective reality lead to distortion. Plantinga and Poythress have shown us how the Christian doctrines of God and its view of man can shed light on the nature of mathematical truth and meaning. As I see it, the battle lines are being more clearly drawn today. We are being enticed to give pure humanistic and secular descriptions of the discipline of mathematics. I find these descriptions most unsatisfying. Perhaps the time is past when the commonly held view of mathematics will allude to the mathematical design and law in the order of things. Yet, we Christians must continue to openly and unashamedly express our views. All the current descriptions have done little to undermine the naiveté, the awe and the wonder of "the mathematical experience."

> Jeremiah 9:23.
> This is what the Lord says:
> "Let not the wise man boast of his wisdom
> or the strong man boast of strength or the rich man of his riches
> but let him who boasts boast about this:
> > that he understands and knows me,"
> > that I am the Lord who exercises kindness,
> > justice and righteousness on earth,
> > for in these I delight, declares the Lord

> For Willem Kuyk
> Isaiah 55:8
> "For my thoughts are not your thoughts,
> neither are your ways my ways," says the Lord
> "As the heavens are higher than the earth
> so are my ways higher than your ways
> and my thoughts than your thoughts..."

<u>Bibliography</u>

[1] Kline, Morris. Mathematics: The Loss of Certainty, Oxford University Press, New York, 1980.

[2] Davis, Philip J. and Hersh, Reuben: The Mathematical Experience, Birkhauser, Boston.

[3] Goodman, Nicholas: Mathematics as an Objective Science, Mathematical Monthly, Vol. 86, No.7, Aug.-September, 1979.

[4] Cohen, Paul J.: Comments on the Foundation of Set Theory in Dana Scott (ed), Axiomatic Set Theory, pp. 9-15, Providence, RI, American Mathematical Society, 1971.

[5] Thom, Rene: Modern Mathematics: An Educational and Philosophical Error? American Scientist, 59, pp. 695-699 (1971)

[6] Gödel, Kurt: What is Canter's Continuum Problem? In P. Benacerraf and H. Putnam (eds.) Philosophy of Mathematics, Selected Readings, pp. 258-273, Englewood Cliffs, Prentice-Hall.

[7] Wolterstorff, Nicholas: On Universals, The University of Chicago Press, Chicago, 1970.

[8] Tol, Anthony: Vollenhoven, Dooyeweerd and Law, Anakainosis, Vol. 3, No.4, June 1981, pp. 7-10. I

[9] Plantinga, Alvin C.: Does God Have a Nature? Marquette University Press, Milwaukee, 1980.

[10] Poythress, Vern S.: A Biblical View of Mathematics, Chapter IX, in Gary North (ed.) Foundations of Christian Scholarship, Vallesito, CA, Ross House, 1976.