Teaching the History of Mathematics Using Architecture and Art

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Background

Point Loma Nazarene University (PLNU) is a Christian university deeply committed to teaching in the liberal arts tradition. As a consequence, all of our students take roughly 64 semester hours of "general education" courses as part of the 128 semester hours required for graduation. One of the challenges of teaching at such an institution is helping students to value the core curriculum and to see that the core "is useful, because understanding is useful" (Whitehead, 1967). We want them to know that ultimately the liberal arts are "essential in making our way through the uncharted waters, the open seas, of our lives" (Bennett, 2003) and thus the core is part of becoming an "educated person."

PLNU is currently evaluating its general education curriculum and asking significant questions about the specific content of the core and the connections being forged between classes in the core and classes in the majors. The university derives its academic life from the philosophy of hospitable conversation held in a covenantal community. This means that we seek to foster a climate of openness in and out of the classroom where the community (students, faculty and staff) can discuss and be shaped by a wide variety of ideas; that the ultimate goal of education is for each individual to incorporate those ideas into their personal understanding of the world. For good expositions of this philosophy see Bennett (Bennett, 2003) and Palmer (Palmer, 1993).

The Mathematical, Information and Computer Sciences Department at PLNU teaches an annual course called "Topics in Mathematics." This two-hour course rotates among the faculty and the structure and content of the course is completely determined by the professor. Our goal is to offer our students an opportunity to learn some aspect of mathematics that is not covered in other coursework. I recently taught "Topics in Mathematics: Mathematics, Art and Architecture." My goal was to help students see, discuss and understand connections between the mathematics that they were studying and some aspects of the general education core that is required of them.

This course, taught as a conversation between ideas, was a great deal of work to prepare because it caused me to reach into areas in which I am not an expert but it was a success and well worth the effort. In this paper I will briefly outline some of the details of this somewhat quirky and highly personal class and also discuss some of the lessons that I learned.

Student Involvement

One of the key commitments of our department is to help our students develop the skills to read, write and talk about mathematics. This means that from their freshmen year we push

them to derive meaning out of written theorems and definitions, write both technical and expository documents and give oral presentations. This course extended all of those skills. The students were expected to read some original documents (in translation), research and prepare written and oral presentations and actively engage in the classroom discourse about ideas that arose from material presented, hands-on activities and field trips. It was expected that all students taking the course were at least second semester sophomores. Thus all participants had studied three semesters of calculus as well as calculus-based physics (mechanics) and were either currently engaged or had already taken course work in linear algebra and proof writing using number theory.

Mathematics Taught

My goal was for students to see connections between mathematics and "the real world", not necessarily to learn new mathematics. Topics studied included notions of symmetry and other concepts from geometry, the mathematics and physics behind various construction techniques and the original highly mathematical explanations of drawing objects in "true" linear perspective.

Architecture

This course grew out of my interest in sacred architecture, particularly churches of the seventeenth and eighteenth centuries. In the summer and fall of 2001, I spent a sabbatical looking at the connections between the mathematical work of Sir Christopher Wren and Robert Hooke and the architecture of the approximately fifty churches that they designed for the City of London after the Great Fire of 1666. As I worked through this project, it became clear that one of the most mathematical tools that Wren and Hooke used was their ability to view each design project as an over-determined, highly parameterized problem. Many of their best known designs involved some very clever compromises to deal with the excess of parameters. Understanding these parameters is the very narrow focus that I used when teaching students about architecture. For a summary of these ideas see Zack (Zack, 2006).

Symmetry

Wren and Hooke were two of the key innovators in the "English Classical" style of architecture (closely related to what is more generally known as Baroque). The churches that they designed were very different from the gothic churches that they replaced. English Classical architecture derives a great deal of its style from fifteenth and sixteenth century Italian architecture which was romanticized as a recovery of the ancient Roman and Greek styles.

Students in the course read several original texts (in translation) including Vitruvius (Roman, first century B.C.) (Vitruvius, 1960), Alberti (Italian, fifteenth century) (Alberti, 1997) and Palladio (Italian, sixteenth century) (Palladio, 1965). These texts had a significant influence on Wren and all three strongly emphasize that beauty is defined by symmetry and other "perfect" proportions (e.g. the golden ratio). The sections of these texts that I used in classroom discussions and presentations examined worship spaces and basilica (not a worship space but one that heavily influenced Christian church design).

This portion of the course sought to help students make connections between geometry and the mathematics of symmetry and the historical knowledge that they gained from their year-long general education course in world civilizations. The careful reading of the historical texts to understand details was another way to help students see that to really understand complex ideas, texts need to be read slowly and repeatedly often with a pencil in hand.

Construction Techniques

Most of Wren and Hooke's church designs incorporated arches. Many also made use of domes, cantilevers and flying buttresses. I divided the class up into groups, gave them blocks and asked them to investigate cantilevers and then make conjectures about the forces involved. The students were also divided into teams and sent off to investigate the physics behind arches, domes and flying buttresses. They compiled their results and were required to give formal group presentations using PowerPoint. I did not give them a great deal of help; instead I encouraged them to talk to their physics professors to clarify what they had studied about vectors and forces. Each student gained a deeper understanding of mechanics (the core physical science required of all math majors) by laboring over these practical problems. This assignment also asked them to prepare clear explanations of fairly complicated problems and their solutions.

Theological Meaning in Church Architecture

In her lovely book *The Geometry of Love*, Margaret Visser (Visser, 2000) uses the example of a small parish church in Rome to discuss the role of the narthex, nave, aisles, apse and altar in conveying theological meaning. Many of the students at PLNU attend non-denominational or "seeker sensitive" churches whose buildings are not constructed in traditional church forms. For most of the students in my class, this information was utterly new.

As part of the course, we took two field trips to local parish churches that are excellent examples of traditional church architecture. The first church is a Roman Catholic Church on the edge of our campus. None of my students had ever passed through its doors. Because it was spring term and we were near Easter, I handed each student an explanation of the Stations of the Cross before entering the church. After discussing the architecture, I allowed them some time to walk the church and to consider what there might be to learn from traditional contemplative practices such as praying the Stations of the Cross.

The second church that we visited is a Greek Orthodox Church in our community. It is a traditional Greek church with icons painted on all of the interior walls and ceiling, a central dome and a rounded apse. None of my students had ever been inside an Orthodox church of any kind. As we entered, I read aloud the Nicene Creed, the creed that is foundational to worship life of all Orthodox communities. I also arranged for one of the priests from the congregation to talk to the students about the use and meaning of both the icons and the architecture of the church. (For a nice discussion on the meaning of icons in the Orthodox tradition see Matthews-Green (Matthews-Green, 2003)).

The time that we spent visiting churches focused both on the parameters involved in traditional church design and connections between those parameters and the information that students had learned in their general education classes in theology. After the second church visit we had a lively class discussion about what it meant to be a member of the Christian family.

Much of it focused on the notion that many of the things that my students had understood to be "idols" might be something different. One student told me that she "had been humbled to realize that there is so much that [she doesn't] know" about other Christian traditions. That one statement made the class a success!

The Work of Christopher Wren and Robert Hooke

Before engaging in a life of architecture, Christopher Wren was well known as an astronomer and a mathematician. He held the Savilian Chair of Astronomy at Oxford and was the Gresham Professor of Astronomy at Gresham College in London. Robert Hooke was the Curator of Experiments for the Royal Society (a post he held throughout his life time) and the Gresham Professor of Geometry. Wren and Hooke were life-long friends and through a complex sequence of royal and civic appointments were tasked with rebuilding the City of London after the Great Fire of 1666. More than 80 churches were consumed in the fire and roughly 50 parish churches and St. Paul's Cathedral were rebuilt.

The parish churches posed very interesting parametric problems. Wren and Hooke were trying to create churches in the "new" English Classical style, yet needed to construct the churches as efficiently and economically as possible. Many of the parcels of land were irregularly shaped and land locked, and the foundations of the old buildings were to be used as part of the reconstruction whenever possible. In the course, I had the students consider a few specific examples by first listing the parameters and then seeking solutions on their own. The churches I used as examples were: St. Stephen's Walbrook, St. James Garlikhythe and St. Antholin, Budge Row. (See Jeffery (Jeffery, 1996) for detailed information on the City Churches.)

After we had discussed the complexity of these problems I showed them Wren and Hooke's solutions. It produced some interesting conversation about the compromises that have to be made in over-determined systems; that in fact the judgement calls needed to find a good solution are as much an art as a science.

Art

Again, the focus here was extremely narrow. I considered the development of painting and drawing in linear perspective. Students were interested to learn that it was a mathematician not an artist (Leon Alberti) who first wrote an analytical description of the needed technique.

The Development of Perspective

I began this section of the course by looking at a great many images of paintings from 1200 – 1800 A.D. This was a "crash course" on tracing the idea of "reality" in image representation through history. I used a standard cannon of well know (mostly Italian and Dutch) art work. My colleagues in the Art Department were very helpful in answering questions; however, any basic book on perspective will have such a representative sample (Field is one such book (Field, 1999)).

Most of the students had seen many of these images in general education courses but had not seen them strung together in this particular manner and viewed so quickly. I focused solely

on the use of perspective not on any of the other details related to the painter and painting. The classroom conversation centered on trying to identify and describe geometrically what made images look realistic or slightly distorted.

Vanishing Points and Tiling

The Public Broadcasting Series *The Day the Universe Changed*, has been issued in video tape (Burke, 1986) and book form (Burke, 1995). In that collection is an episode titled *Scientific Imagination in the Renaissance* that tells the story of the development of perspective beginning with Brunelleshi's lost painting of the Baptistery in Florence. The accompanying book by James Burke, has a clear exposition of these ideas in the chapter titled "Point of View."

After viewing the tape, I had the students look at Alberti's description in *On Painting* (in translation) (Alberti, 1966) of the vanishing point and the grid lines needed to draw a tiled floor in perspective. The students struggled with reading this highly technical document. There are however some helpful end notes provided by the translator that contain further explanations and useful diagrams. After a significant amount of labor to understand Alberti's method, the students successfully drew a tiled floor in accurate perspective. This had them work out with their hands the geometric ideas that we discussed when viewing paintings and allowed them to know kinesthetically the deep connection between (representational) art and mathematics.

Lessons Learned

This course involved a significant amount of work to prepare and teach, however I consider it a successful experiment for several reasons.

- I stretched myself and modeled for my students a willingness to venture into the unknown and learn new things together. This opened the door for a different kind of classroom discourse. Parker Palmer says "[W]hen a teacher is continually exploring alien, uncharted territory, humility and openness are cultivated. That teacher is constantly reminded that he or she does not know it all, and the resultant openness of mind creates a space where both students and subject can speak a fresh truth."
- Students were excited to see connections between their major and the general education core. This did foster some good conversations about the liberal arts tradition and why I believe it is what makes people "educated" for life rather than just for a profession.
- The next time I teach this course, I will make a greater use of original texts and allow students more time to struggle over them. Because the material was unfamiliar to us all, I was too ready to help students in my office hours when they were struggling with the language rather than sending them off to work on it themselves for several days before coming to me.

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