Connecting Mathematics Students to Philosophy and Faith

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Introduction

Many Christian colleges and universities place a high value on the continuing pursuit of integrating faith and learning. Faculty are encouraged and supported to seek out meaningful connections between their Christian faith and their scholarly discipline. The motivation behind such an effort is clear: to eliminate the ingrained notion that academia and faith provide mutually exclusive pathways to travel in the pursuit of truth. Exploring the rich supply of integrative questions that arise in each discipline is certainly a noble calling, but also brings its challenges. This is especially true in the area of mathematics.

One of the ongoing challenges that we face as educators is how to bring our Christian faith into the mathematics classroom to engage students in this important conversation. I propose that quality, practical examples of successful classroom integration projects will serve to stimulate innovation and creativity within the body of Christian mathematics educators. Toward that goal, I will discuss a class project that I implemented in a Real Analysis course. The idea for this project grew from my involvement in a workshop called *The Vocation of the Christian Professor* in the summer of 2010 at Whitworth University. This workshop challenged me to look deeper at how my faith shines light on my foundational assumptions of mathematics. Thus, the class project makes use of mathematical philosophy as a means of facilitating faith integration.

Project Motivation

The types of questions one asks about mathematics determines the degree to which meaningful integration can take place. I suggest there are three main questions that represent different levels of engagement with mathematics.

- *How* do we do mathematics?
- *Why* does mathematics work?
- *What* is the nature of mathematics?

The vast majority of people are satisfied to engage mathematics at the level of the *how* question. How do I get a common denominator? How do I factor a polynomial? How do I take a derivative? With my students' current mathematical knowledge, this surface level engagement produces very little prospects of significant faith integration. Mathematics is viewed as a tool for science and a means of understanding the world. Thus, the development of specific skills is the emphasis of mathematics education from kindergarten through high school.

It is often not until college when mathematics majors have the opportunity to delve into the *why* question. Here, the engagement with mathematics is at the level of proof, where students come to understand the rigorous and logical process by which mathematics is produced and justified. A large and increasing skill set must also be accompanied by a deeper conceptual understanding of mathematical objects and the interrelations that exist between them. It is in this understanding that students can begin to probe the deeper integrative questions. For this reason, the project was implemented in a proof-based upper-division course.

The fruitful integration discourse that this project sought to produce could occur only at the deepest level of engagement with mathematics: the *what* question. What is the nature

of mathematical objects? Is mathematical truth created or discovered? Is mathematics a part of the structure of the universe or the product of the human mind? How do we account for the effectiveness of mathematics in describing the physical universe? These questions, among others, serve as a gateway between mathematical, philosophical, and even theological concepts. This is fertile ground for students to begin to ask themselves the tough questions and grapple with the interplay between their faith and their view of mathematics.

Class Project

The project itself is not intended to be an exhaustive in-depth study of mathematical philosophy. Its purpose, however, is to provided students an *overview* of philosophical questions to stimulate conversation and reflection. The project took place during the first semester of a two-semester sequence of Real Analysis consisting of 25 students; however, I believe it would fit well within any proof-based upper-division mathematics course or a senior capstone course. It spanned roughly three weeks of the entire semester, but only half of two class days were taken up with discussion time. Thus, I was able to minimize the amount of class time taken away from covering the required topics of the course.

There were three primary goals I sought to achieve through this project. By the end of the semester, I wanted students to:

1. Be exposed to deeper questions about the nature and foundational assumptions of mathematics;
2. Gain a basic understanding of the four main philosophies of mathematics from a historical perspective;
3. Reflect on these questions in light of their own Christian faith or worldview.

The first goal is intended to simply broaden the students view of mathematics. That is, to help them see and appreciate an area of mathematical understanding beyond the standard undergraduate curriculum. I believe that wrestling with these deep questions is a worthwhile goal, especially at a liberal arts institution where a high value is placed on developing students with strong intellectual capacity and curiosity.

The second goal is necessary in order for students to grasp how others throughout history have addressed these questions. They have the opportunity to examine the mathematical philosophies of logicism, intuitionism, formalism, and platonism as well as the mathematicians who espoused them. This provides students a framework of terminology and concepts to help process many of the deeper issues addressed.

The third goal gets at the heart of the faith integration component by allowing students to ground their philosophical viewpoints upon their personal faith or deep convictions. I was intentional in my use of the word "worldview" in this goal for two reasons. First, like many Christian institutions, Whitworth University's student body consists of both Christians and non-Christians. Therefore, the concept of a worldview is broad enough to apply universally to all students. Second, prior to this class each student had already spent a significant amount of time defining and reflecting upon his/her own personal worldview. Whitworth has a worldview studies program that is part of the general education curriculum. It requires that students take a sequence of three courses (Core 150, Core 250, and Core 350) that furnish them with the basic categories of worldview thinking, including the nature of God, the nature of humanity, how we know, the nature of reality, and how we should live individually and corporately. The students are also equipped to explore the parameters of their own worldviews. I utilized Whitworth's Core program to provide a foundation for each student to view the deeper questions of

mathematics.

The project itself is broken up into three separate assignments. The first two consist of article readings, a short response paper, a time of small group discussion, and a brief whole class discussion. The third assignment, a culminating paper, requires students to personally reflect on the first two assignments.

Assignment 1

Students read *Mathematics as an Objective Science* (Goodman, 1979), *Mathematics: A Concise History and Philosophy* (Anglin, 1996, chapter 39), and *Mathematics and the Myth of Neutrality* (Bishop, 2001). The first two articles provide a historical context for the development of philosophies regarding the foundations of mathematics. They explore how formalism, intuitionism, logicism, and platonism developed, define their core tenets, and discuss the inadequacies of each in providing a comprehensive foundation for all mathematical thought. The third article's thesis is that beliefs are integral to mathematics and that ultimately mathematics is not neutral, but shaped by worldviews. This viewpoint is articulated in the article by the following quote from Paul Ernest (Bishop, 2001):

> *Mathematical truth ultimately depends on an irreducible set of assumptions, which are adopted without demonstration. But to qualify as true knowledge, the assumptions require a warrant for their assertion. There is no valid warrant for mathematical knowledge other than demonstration or proof. Therefore assumptions are beliefs, not knowledge, and remain open to doubt.*

After reading the articles, the students wrote a short response in which they summarized and briefly reflected upon the content presented. Then, they had time during class to get in groups of three or four to discuss what they had read. To guide the discussion forward they were given a list of thought-provoking and reflective questions. The following is a list of the questions posed:

- Do you agree or disagree with Ernest's quote above? Explain.
- To which of the four philosophies would a statistician, geometer, or physicist likely belong?
- How would each of these philosophies likely view the Fundamental Theorem of Calculus?
- We have discussed the four ways of knowing: intuition, empirical senses, innate reason, and authority. To what extent has each of these played in your understanding of mathematics?
- Consider the definition of a function given in your book. List out all the definitions and operations that must be known prior to understanding the concept of a function. How is each of these things known?

One of the challenging aspect of developing this project was coming up with good questions that stimulate reflection and discussion. For this assignment, the questions were instrumental in forcing the students to think deeply about their understanding of mathematics. When confronted with the fourth question in the list, many students initially indicated that authority was their primary mode of understanding mathematics. The teacher and the book tell us what mathematics is, and we believe it. Yet, the other epistemologies begin to take a more significant role as the students begin to wrestle with their dependence on intuition and reason.

The purpose of this assignment is to begin to move students into a deeper conversation about mathematics, one which many students had never taken part. It addressed the goal of exposing students to the larger philosophical nature of mathematics. While there was no explicit Christian focus to this assignment, it laid the groundwork for what was to come. It forced students to confront the idea that one's philosophical view of mathematics may be shaped by a particular set of beliefs and have an impact on the ways in which one engages with mathematical concepts. Several students commented that this philosophical discussion was entirely new to them. It served to enlarge their view of mathematics beyond simply calculations and proofs.

Assignment 2

This assignment is primarily designed to explore how a Christian perspective can inform this philosophical discussion. With this understanding, the students read two articles in the Journal of ACMS entitled *An Augustinian Perspective on the Philosophy of Mathematics* (Bradley, 2007) and *Mathematics as Worship* (Stucki, 2001). The first article examines how Augustine addresses four basic philosophical questions of mathematics based upon his view of God and scripture. What is the nature of mathematical objects? How do we obtain knowledge of them? What is the meaning of "truth" in mathematics? How do we account for the effectiveness of mathematics in describing the physical universe? Augustine's views are presented in a very accessible way for students to grasp. This provides them a genuine example of relating the Christian faith to mathematics. As Bradley states,

> *The Augustinian view of mathematics has much to commend it. It's inspiring - from this perspective the capacity to do mathematics is a gift from God, its content originates with God, a mathematical career is a calling to discover God's wonders, and it leads both to service of his kingdom and to worship.*

Bradley goes on to describe the historical movement toward secularization and the diminishing role of the Augustinian perspective. In Stucki's article, worship is defined broadly as "a lifestyle of submission, in obedience to God." In this context, he explores how mathematics can be viewed as part of worship, the primary and necessary response of man to his creator.

Similar to Assignment 1, the students are given class time to converse about the articles in small groups. Here is a list of the discussion questions posed:

- What can you infer about the worldview of Descartes, Kant, Russell, and Augustine based on their view of mathematics?
- Do you agree with the quote "All knowledge is contingent upon faith"?
- What was the relationship between theology and mathematics before the Renaissance and Scientific Revolution? After?
- How does the author define worship in this article? How does he claim that mathematics can be seen as worship? Do you agree?
- What do you think about the statement "God created the integers and all else was the work of man."?
- Consider the following quote by Hilda Hudson: "To all of us who hold the Christian belief that God is truth, anything that is true is a fact about God, and mathematics is a branch of theology." In what ways can mathematics be classified as a science? In what ways can it be classified as a branch of theology?

The students seemed quite engaged during this discussion time. I saw more lively debates and open conversations among students when matters of faith are interjected due to the

more personal nature of the topic. This is where many of them began to see some meaningful connections between their faith and their view of mathematics. The goal of the next assignment is to convert that insight from the verbal discussions into a reflection paper.

Assignment 3

The project culminates in a two-part paper that brings together many of the concepts developed in the articles and discussions. First, in a two-page paper the students were to articulate a personal answer to one or several of the philosophical questions introduced in class regarding the nature of mathematics. Their response was not simply to draw from what they thought or felt, but from their deep convictions about the nature of God, humanity, and the world. Toward that aim, they were prompted continually to seek out connections between their view of mathematics and their personal faith or worldview. They were encouraged to use scripture, if they wished, as they reflected. I also made available some supplementary articles that were not required reading but encouraged for additional resources. Those articles, given in the list of references, include Bishop (1996), Fackerell, and Hampton.

The paper prompt was intentionally vague in order to provide freedom for the students to explore what they wanted. I assured my students that there were no right or wrong answers. These questions had been pondered for centuries with no tangible, concrete conclusions. They were being graded not heavily on content, but rather on the depth of reflection and the connections between their worldview and mathematics. This is why the foundation of Whitworth's Core program was vital to the students' ability to engage in this way. Already confident of their personal view of God, humanity, and the world, they were to examine the nature of mathematics in light of their beliefs.

The second piece of the paper was designed to bring some practical application to the philosophical rumination. I wanted students to discuss how any of this makes a difference in their lives. Thus, they wrote a one-page paper in response to the following question: how does your view of mathematics, as described in part one of the paper, affect the way you do mathematics? They were to reflect on their own motivation for being a mathematics major. They were also prompted to consider what differences might exist in mathematics from a Christian perspective and from other perspectives.

This part of the paper was primarily intended to move students away from the standard response of "I like math because I'm good at it." After Whitworth, many of the students would go on to graduate school, teach, or work in industry in some field related to mathematics. This required them to dig deeper and examine how God might use mathematics in their lives and their future vocation. Many students enjoyed the opportunity to write about mathematics on a more personal level.

Student Feedback

At the beginning of the semester I asked each student if they had previously given any thought to the philosophy or origins of mathematics. All but one student said that they had given very little or no thought to this subject. So the material covered in the article readings and discussion time was uncharted territory for nearly all of the students. Overall, they responded very positively to this project. I received feedback from the students both informally through conversations and formally through a survey I conducted at the end of the semester. Following are some of their responses:

> *It was surprising that there were so many different views of the origins of mathematics. Before this assignment I believed that everyone had the same view about where*

mathematics came from.

I feel as if I have more tools to articulate my view of mathematics. My view hasn't changed, but I feel better prepared to explain and/or defend my view. It is more defined for me now.

Reading those articles really caused me to think more about what I agree with and what I disagree with and how I would fit into different categories.

I do mathematics because its something I enjoy and its a way for me to achieve my higher purpose of reaching the youth by building personal relationships in the classroom and showing them the beauty of mathematics. I will present some of these same things we talked about to my high school classes to get them thinking about what math really is.

I don't think my worldview impacts how I do math at all. However, it definitely impacts why. The reasons I do math derive straight from my reasons for doing anything which boil down to worldview.

At a minimum, this project served to expand the view of mathematics in the minds of the students. It provided a categorical framework for them to define a view of mathematics that they had never considered. It confronted them with the prospect that their beliefs may meaningfully interact with their view of mathematics. It required them to consider introspectively their motivation and calling as it relates to engagement with mathematics.

From my experience with this project, students are hungry to discuss these foundational areas of mathematics given the opportunity. In fact one of the most frequent comments I received from students was the lack of time committed to discussion. They wanted to hear the viewpoints of others in class beyond their small discussion groups. Although I allotted the last five minutes of the period to a full class conversation, it was not enough. In the future, I will either provide more scheduled time for a large group discussion or have the students engage with several different small groups to allow for more sharing of a diversity of opinions. Another idea that may be beneficial is utilizing an online discussion forum where students are encouraged to post their thoughts.

Discussion

I believe that mathematical philosophy can be a tremendous facilitator of integrating faith and learning in the mathematics classroom. Understanding the underlying assumptions of mathematics in light of a Christian worldview provides a depth of education that a standard undergraduate curriculum may not afford. This serves to educate the entire person, mind and heart, as is consistent with the mission of Whitworth and many Christian liberal arts institutions.

As mentioned earlier, this project relied heavily on the previous work students had done reflecting on their personal worldviews in Whitworth's Core classes. It is vital for them to know exactly what they believe and why they believe it. Yet, this project can still function well at institutions that do not have a similar general education requirement. I would suggest spending a day in class or assigning some additional outside reading that introduces the students to the basic categories of worldview thinking. Once students have shored up their own views on the nature of God, humanity, and the world, they will be better prepared to approach the philosophical questions of mathematics.

Although I felt that my students were prepared to take part in this project, I was realistic in what I could expect from their papers. I was not looking for fully developed philosophical

arguments about the nature of mathematics. Since this was the first time my students had ever confronted these issues, I was satisfied with evidence of self-reflection, critical thinking, and an effort to make worldview connections. The questions that arise from a deep philosophical examination of mathematics may never have satisfactory answers. Yet, we should not let that deter our students from wrestling with them. Their search for answers still has the potential to yield much fruit, including an awareness and clarity of personal vocation. This is vital in moving students beyond the idea that mathematics is simply a tool necessary for a successful career. It also helps students view their mathematical skills and abilities for greater purposes in light of their calling.

Conclusion

Mathematics is a discipline that, on the surface, yields little or no prospects for a meaningful discussion of faith integration in the classroom. It works universally and consistently without regard to one's religious, moral, or ethical predispositions. Therefore, it takes some focused effort to dig down to a place where mathematics and faith come together. Fortunately, through the ACMS we are blessed with a community of fellow diggers and a tremendous supply of resources. In this article I provide an example of how to effectively utilize those resources in the classroom. The project described in this article is simply a starting point for me. I am sure this is only one of many ways to successfully bring students to a place of meaningful interaction with these important philosophical issues.

I believe there is currently a lack of awareness and acknowledgement of mathematical philosophy among undergraduate mathematics majors. Gaining insight into these philosophical questions may not help students get into graduate school or find a job, but it certainly enlarges their view of mathematics. It also provides a means of deepening the important discussion of the interactions of faith and mathematics. At Whitworth University, and many other Christian liberal arts institutions, this type of discussion is highly valued as a key component of a larger goal of integrating faith and learning.

References

Anglin, W.S. (1996). *Mathematics: A Concise History and Philosophy*. New York, NY: Springer Verlag.

Bishop, Steve (1996). Beliefs Shape Mathematics. *Spectrum*, 28(2).

Bishop, Steve (2001). Mathematics and the Myth of Neutrality. *Christian School Education*, 5(4).

Bradley, James (2007). An Augustinian Perspective on the Philosophy of Mathematics. *Journal of the Association of Christians in the Mathematical Sciences*, 2006-2007 Issue, http://www.acmsonline.org/journal/2007/Bradley-Augustinian.htm.

Fackerell, Edward (2002). The Relationship Between Mathematics and the Christian Faith. *Christian Teachers Journal*, 11(2).

Goodman, Nicholas (1979). Mathematics as an Objective Science. *The American Mathematical Monthly*, 86(7).

Hampton, Charles (1977). Epistemology to Ontology. *Journal of the Association of Christians in the Mathematical Sciences*, 2004 Issue, http://www.acmsonline.org/journal/2004/Hampton77.htm.

Stucki, David (2001) Mathematics as Worship. *Journal of the Association of Christians in the Mathematical Sciences*, 2004 Issue, http://www.acmsonline.org/journal/2004/Sucki01.htm.