Math History Study Abroad Program: Learning Math History in a Cultural Context

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1. Introduction

In January 2011 fifteen Whitworth University mathematics students and I, their professor, traveled through Europe to study the history of mathematics. The goal was to gain an understanding of how mathematical ideas have developed over time: how social, cultural and historical factors have influenced the development of mathematics and conversely, how mathematics contributed to society and human culture. Over a course of three weeks we traveled to three countries and over a half dozen cities, viewing the tools, papers and workbooks of these mathematicians, seeing their engineering and artistic creations, and learning from local experts as they guided us through science museums and archives. The experience helped us to understand the broad roles mathematics and mathematicians played in the world around them.

The journey actually began several months earlier during a Fall Prep Course. Readings from the textbook, *Mathematics in Historical Context*, by Jeff Suzuki provided a good foundation for understanding how mathematics is an integral part of culture. The book gives a broad overview of the history of mathematics from the Ancient World (Prehistory, Egypt and Mesopotamia) to post-WWII. Though we concentrated mainly on the last seven centuries, we also studied some Greek, Egyptian, Babylonian and Islamic mathematics, in anticipation of seeing some of the artifacts from these civilizations in museums during our travels. Movies about Roman engineering and biographical films on Galileo, Newton and Einstein helped us to visualize what the world must have looked like when these mathematicians made their important contributions. As part of the fall course, each student chose a particular mathematician to research. The students wrote a term paper describing their mathematician in terms of historical and cultural context in which he/she lived, as well as their contribution to mathematics. These mathematicians correlated with various sites we visited during our travels. Students presented on the life of their mathematician at the corresponding site. Students learned in-depth about a mathematician through a combination of library research and then actually seeing where that mathematician lived and worked.

2. Rome: Engineering, Art and Politics

Our travels abroad began in Rome where examples of ancient Roman engineering surround you. These engineering feats played a large role in Rome's ability to grow as an empire. For example, water was abundant in Rome thanks to an aqueduct system designed using arches, a slight grade and gravitational pull. Eleven aqueducts provided Rome with enough water to sustain a population of one million people. This abundance of water, which made regular bathing possible, also contributed to the Roman's worldview that they were superior civilization because they were cleaner than other cultures.

(a) Roman Fountain

(b) Santa Maria degli Angeli, central hall of the former Baths of Diocletian

Figure 1: Abundance of water in Rome

Using his access to water, Nero built a huge lake in the center of Rome as part of his palace complex to demonstrate his wealth and power. After Nero's death, the Emperor Vespasian drained the massive lake Nero had built (for his personal pleasure) and replaced it with the Coliseum – built for the people of Rome. Even though we had watched a film about the building of the Coliseum, we were still in wonder when we actually saw the ingenuity and intricacies of its design. These included assigned seating, an awning/weather protection system, and layers of subfloor with elevators that allowed wild animals to seemingly spring out of nowhere onto the amphitheater floor. The Roman Forum, for centuries the center of Roman public life, is located next to the Coliseum.

Figure 2: Roman Forum

Commerce, the administration of justice, and religious activities of Rome were conducted at the Forum. It was also the site of triumphal processions and a venue for public speeches. Visiting the Forum and Coliseum gave us a real sense of the grandeur and power of Rome and the role engineering played in making it possible. We continued to see the influence of Roman architecture and engineering in the other cities we visited, especially Florence and the sites in England.

We spent four days in Rome. We learned about life in an ancient commercial city through our visit to the ruins of Ostia Antica, a Roman port town, where we walked through the streets of the city, saw the design of amphitheaters and temples, public latrines, burial grounds, homes and marketplaces. Mathematical patterns were reflected in the geometric designs of floor tiles as well as the layout and design of the city.

(a) Ostia Antica floor tiling

(b) Amphitheater in Ostia Antica

Figure 3: Ruins of Ostia Antica

We continued to see the role mathematics played in aesthetical design when we studied piazzas and architecture in Rome. Piazzas were public spaces and the design of these piazzas reflected both the political and the mathematical landscape of their creation. An example is the Piazza del Campidoglio in Rome, designed by Michelangelo to reflect the civic grandeur of the governing seat of Rome. Starting with two awkwardly placed palaces and a muddy hill, Michelangelo used symmetry and an oval shaped design to create a harmonious trapezoidal space. The orientation of the plaza faces away from the ancient center of power in Rome, the Forum, towards St. Peter's, emphasizing the city's dependency on papal power. Alberti's writings on proportional prescriptions for urban spaces and Brunelleschi's linear perspective also influenced the design of piazzas. The design and construction of domes, such as the domes of the Pantheon and St. Peter's (and the Duomo in Florence) were also dependent on mathematics. The pre-trip readings and films provided the students with background information regarding the engineering of these domes. However, actually seeing the size and massiveness of these domes created a deeper appreciation of these engineering wonders. The students realized that mathematics was intricately woven into the everyday lives of the people and that engineers, architects and artists were also mathematicians. Mathematics was sometimes used in the construction of art and sometimes the art itself reflected the important role that mathematics played in the culture. For example, Raphael's painting, *The School of Athens,* in the Vatican Museum in Rome, portrays an idealized community of intellectuals from the entire classical world. In the picture you can see

Pythagoras with his perfect numbers, and Euclid demonstrating some geometric proposition with a pair of compasses upon a slate. Raphael's vision of Humanism pays tribute in *The School of Athens* to the importance of mathematics in human learning and understanding.

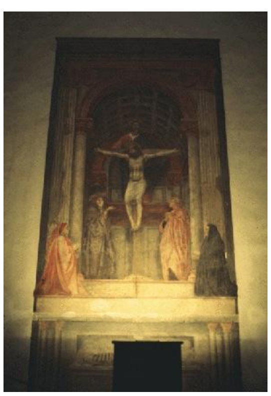
(a) Piazza del Campidoglio, Rome

(b) Euclid demonstrating a mathematical proposition in Raphael's *School of Athens*

Figure 4: Mathematics in Architecture and Art

3. Florence: Renaissance Art and Science

From Rome we journeyed to Florence, the heart of the Italian Renaissance, home to Galileo, Leonardo DaVinci, Michelangelo and many other Renaissance artists and scientists. Florence provided the opportunity to learn about these key figures in the context of where they lived, worshiped and did some of their greatest work. Mathematics played a major role in architecture and art during the Renaissance, especially with the invention of mathematical linear perspective. This innovation is credited to Brunelleschi, a sculptor, architect and artisan-engineer. We saw many examples of linear perspective applied to art and architecture in the church of Santa Maria Novella including Masaccio's 1428 fresco of the Trinity, credited as the first painting in the history of art to use perfect linear mathematical perspective.



(a) Masaccio's *Trinity*

(b) Brunelleschi's dome

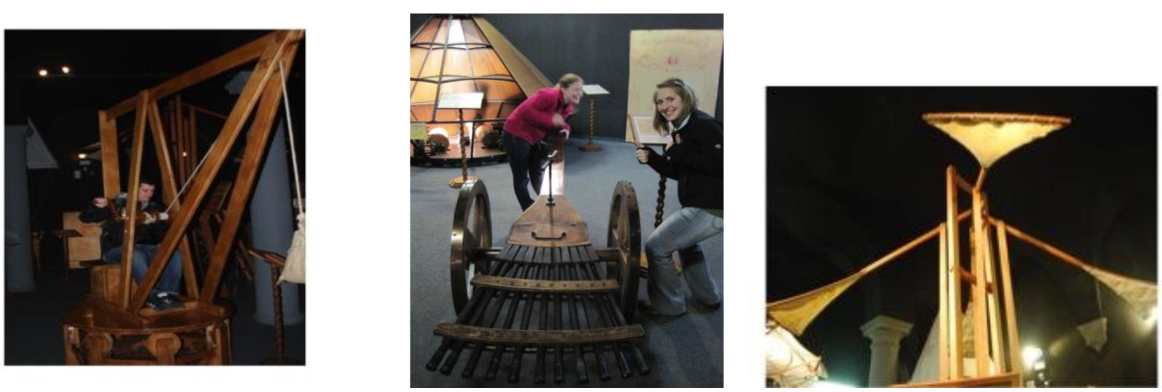
(c) Giotto's Tower relief

Figure 5: Sample of Florence art

Brunelleschi is also considered the father of Renaissance architecture with its emphasis on symmetry, proportion, geometry and the regularity of parts. We saw examples of Brunelleschi's architectural achievements in Florence in the church of San Lorenzo with its Corinthian columns and geometric balance and harmony, and in the Duomo, Florence's cathedral. Construction of the Duomo began in 1296 using Cambio's design. Cambio envisioned a dome for the church even though he, nor anyone in Italy at that time, had any idea of how to construct it. But he believed that by the time building of the cathedral had progressed to the point that the dome was to be built, God would provide a man with the mathematical skill to build it. That man was Brunelleschi and during their prep course the students read Ross King's book, *Brunelleschi's Dome*, which tells the story of the building of the dome. In Florence the students now climbed this dome - the largest masonry dome ever built. They viewed the unique herringbone pattern construction of the bricks which permitted the dome to be built without a support system or scaffolding.

Across the street from the Duomo Giotto's Bell Tower stands with its allegorical reliefs depicting astronomy, medicine, the building art, weaving, navigation, geometry and arithmetic. These decorations recount the destiny of man, from his creation to his dominating the world by learning technology. One relief depicts Gionitus, the mythical inventor of Astronomy, observing the height of celestial bodies using a quadrant.

The Italian region of Tuscany, which includes Pisa and Florence, was Galileo's home, and his work is celebrated and memorialized in the Museo Galileo in Florence. A science guide led us through the museum, demonstrating Galileo's instruments (the museum houses Galileo's original instruments). In addition, we saw barometers, telescopes, quadrants, chemical flasks and an all-in-one laboratory table. Our guide explained the role science and mathematics played in creating the Scientific Revolution, changing the way people viewed and understood the universe. We developed an appreciation of the variety and extensiveness of the scientific explosion that was taking place across Europe in the 17^{th} and 18^{th} centuries. A day trip out to Pisa provided further opportunities to view important Galileo sites. We visited the Tower of Pisa, where Galileo supposedly conducted his experiments on gravity, and saw the chandelier in the Duomo of Pisa whose swaying led him to an understanding of pendulum motion.



(a) Crane (b) Machine gun (c) Flying Machine

Figure 6: Models of Leonardo da Vinci's inventions

Art, architecture, science, politics, inventions, and mathematics were closely intertwined in the Italian Renaissance, and no one reflected this collaboration of disciplines more so than Leonardo Da Vinci, the archetypical Renaissance man. At the Leonardo Museum in Florence we tried out several models of Da Vinci's inventions. The variety of these models showed the span of Da Vinci's interests and ideas. Many of these inventions functioned as practical work machines. Da Vinci also designed war machines; he understood that rich rulers were willing to support mathematicians and inventors if these inventions served their military interests. Yet others of Leonardo's inventions revealed his visionary and somewhat fanciful designs for flying machines, floats for walking on water and even diving suits for underwater exploration.

4. London: British Royal Society and the Scientific Revolution

The Scientific Revolution may have begun in Italy with Galileo's astronomical observations that supported the Copernican heliocentric universe, but it really took off in England in the scientific community of the British Royal Society, early promoters of the scientific method. A visit to the Royal Society headquarters in London, with a guided tour by the head librarian there, introduced the students to this influential group of individuals who had changed the way science was done: from talk and conjecture to verification by experiments. We saw original scientific papers written by Royal Society members, including Newton's Principia (accompanied by Newton's drawings and cartoon sketches). Other Royal Society items on display were Boyle's air pump, Newton's telescope and Davy's mining lantern.

We saw examples of the influence of Royal Society members in navigation during a day trip out to the Royal Observatory and Maritime Museum in Greenwich. Among these was the Harrison clock, the clock that solved the longitude problem for navigation.

Figure 7: Newton artifacts at the British Royal Society in London

During the English Civil War many members of the Royal Society worked at Oxford as a part of a scientific research group. A science tour of Oxford along with a visit to its Museum of Science provided many examples of their work. As part of a workshop at the Museum of Science the students learned how to use an astrolabe and the way people used it to make sense of their universe. The students also ways scientific instruments were used to tell time and determine navigational position, and how the development and improved design of these instruments allowed for a better understanding of the workings of the universe.

Oxford also provided an opportunity to see one of Christopher Wren's earliest applications of mathematics to architecture in his design of the Sheldonian Theatre. In London, St. Paul's Cathedral, considered to be Wren's greatest architectural achievement, showcased his mathematical training. The design of the cathedral contains many mathematical elements, including a geometrical staircase and the different mathematical curves that determine each of its three domes. Wren designed the outer dome to be spherical in shape, appealing from a distance but also tapping into the idea of the church representing the spherical shape of the cosmos.

Figure 8: The three domes of St. Paul's Cathedral in London

He based the innermost dome, seen from inside, on a catenary curve. This design serves a dual purpose, first of an optical illusion – it draws the eye upward, making the dome seem higher than it is, but also provides a real structural advantage since a upside down catenary arch supports its own weight with pure compression and no bending. Finally, dome between these two, is conical in shape. It is based on the curve $y = x^3$, which Wren and his fellow BRS friend, Robert Hooke, thought to be the perfect shape for a dome.

Figure 9: Wren church in London

The insides of Wren's churches also reflect the changing worldview of the Reformation with regard to the role of the church building. Wren designed his churches to reflect his view that a congregation should be able to see and hear all that went on in the church. His churches are airy, with windows containing clear glass, so people could have light to read the Scriptures. He also often brought the altars in his churches forward towards the congregation so they could see and hear the liturgy better.

London provided opportunities to view mathematical artifacts from many different periods of history. At the British Museum in London we were given a behind the scenes viewing of the Rhind Papyrus, which is too delicate to be put on display in the museum galleries. At this museum we also saw examples of Greek architecture, Babylonian mathematical texts, and clocks (important for telling time in a city as well as for navigation) from various centuries. There was also a whole wing devoted to the 18th century Enlightenment period, containing exhibits of mathematical and scientific discoveries of this period.

5. Berlin and Goettingen: Centers of 18th and 19th Century Mathematics

Berlin and Goettingen became centers of European mathematics in the 18th and 19th century and Euler, Gauss, Leibniz, Cantor and Riemann were some of the mathematicians that the students learned about during their time in these two cities. Studying mathematics in a historical context led to the students' deeper awareness of the interplay between mathematics and politics. In Rome students saw the way Roman engineering and architecture reflected the worldviews of their rulers. Florence brought further examples with Leonardo da Vinci's war machines, and Galileo's political and religious problems. In England the students heard how the members of the Royal Society were caught up in the politics of the English Civil War and that the Society itself was formed under the Restoration of Charles II. However, at the Euler Archives in Berlin, the students expressed surprise when they learned from Euler's papers that he was often involved in politics. The Professor of History and Science at the Berlin Academy of Sciences translated for us many of Euler's letters. These letters contained many interesting stories, including ones about the occupation of Berlin by Russian troops during the Seven Years War. One letter contained Euler's complaints to Frederick of Prussia regarding the "borrowing" of some of the horses from his Charlottenburg estate by Russian troops. Euler demanded compensation and apparently was generously rewarded. Other letters detailed the many administrative duties, ranging from juggling budgets to overseeing greenhouses, which Euler assumed as the informal director of the Berlin Academy.

Figure 10: Euler’s papers and Leibniz’s calculating machine (Berlin Academy of Sciences)

A day trip out to Goettingen followed our visit to the Berlin Academy of Sciences. Students learned about the mathematicians who lived and worked in both these cities and the reasons behind the rivalries between these two centers of mathematics. Goettingen buildings were littered with the names of famous mathematicians and scientists who lived or worked there. Gauss was one of these mathematicians. He apparently enjoyed the small university town feel of Goettingen as it provided him the opportunity to mix with colleagues from variety of disciplines, or keep his distance, as he pleased. Gauss spent much of his time in his observatory, and we had an opportunity to tour this observatory and to see both the chair in which he sat and the telescope he used as he did his observations. Weber also lived in Goettingen and a statue of Gauss and Weber commemorates their collaboration on the invention of the electromagnetic telegraph (1833). They designed the system to link their places of work. The statue shows the scientists talking about their joint work with Gauss holding a wire (not preserved) in his right hand, its coil at his feet, while Weber’s left hand rests on the telegraphic transmitter.

(a) Gauss’s telescope

(b) Weber and Gauss statute

Figure 11: Treasures of Goettingen

Euler's life in Berlin, where he was involved in all aspects of life around him, including household accounts and everyday housekeeping needs of the Berlin Academy, contrasted with Gauss's "quieter" life in Goettingen, where he could choose the degree of his involvement in matters outside his mathematical interests. Students saw that they were two very different personalities though each a pillar in German mathematics.

6. WWII Changes the Mathematical Landscape

In the 20th century politics again played an integral role in the mathematical landscape. The persecution of the Jews in the 1930's and 1940's resulted in the migration of many prominent Jewish mathematicians from Germany to the United States. Many of them were applied mathematicians, and a new era of applied mathematics resulted in the U.S. Jewish mathematicians who remained in Germany lost their jobs, and sometimes their lives. Readings that students did about this persecution and its ensuing hardships took on new meaning when they visited the Jewish Museum in Berlin and when they saw the many memorials around the city, testifying "Never again".

(a) Jewish Museum (b) Holocaust memorial

Figure 12: Jewish museum and holocaust memorial (Berlin)

While many mathematicians were being displaced in Germany during the 1930s and 1940s, in England, mathematicians, along with linguists, classicists and "anyone good with puzzles" were being recruited to the British code-breaking facility in Bletchley Park. A Bletchley Park Educational Staff member gave us a tour of the park including the rooms where Alan Turing's Bombe machines deciphered Enigma codes. This staff member explained the mathematics behind the Enigma code and showed students how to encrypt and decrypt a message using a real Enigma machine. Students also learned more about the person of Turing, who went from an outcast at school to finding a home in the intellectual camaraderie of Bletchley Park. Turing's ability to think outside the box, challenging previous assumptions about machine learning, led to his invention of the Bombe and the Turing Machine, crucial for code breaking. A computer science museum on the grounds of Bletchley Park commemorates the work of Turing and other computer scientists. It also contains many examples of earliest computers. For the computer science students on the trip, this was a highlight!

(a) Main building

(b) Enigma encryption

(c) Turing Bombe

Figure 13: Bletchley Park: England's WWII Cryptography Centre

7. Students' Work and Reflections

In conjunction with our travels, students had daily reading assignments, which gave further background to the time periods and sites we visited. Many of these readings were original source readings, writings of the mathematicians whose legacies we viewed. As we visited the site corresponding to a student's biographical research, that student gave an oral presentation about his/her mathematician. Hearing about the mathematician and then seeing where they lived and instruments they used, contributed to a fuller appreciation of their lives and mathematical contributions. Students reflected on their experiences through daily journaling. They shared about their experiences with family and friends back home through a blog set up on our university website. Each student was assigned a day to blog and was expected to give not only an account of that day's activities, but a historical context to the mathematics encountered. These blogs had an active following back home and generated interest among students back on campus to become part of the next Math History Study Abroad program. You can read the students' blogs and hear about our adventures from their perspectives at http://ma396.blogspot.com/.

When we returned to Whitworth, I asked the students to reflect on how their worldview had changed as a result of their experiences abroad. Their comments revealed their deeper appreciation for history and culture. Typical student comments were: “Traveling for this study abroad trip opened my eyes to different cultures and brought the mathematicians to life. As we traveled we also learned a lot about the places we were traveling that helped me to see history from a different perspective;” “I was amazed at how much history is in other cultures. It was cool to walk around a corner and see ruins or an ancient statue or a huge cathedral. There is a lot more to the world than our (American) history and my perspective and worldview;” “Study abroad really opened my eyes to different cultures ”; I got to experience new food, new languages, and met new people. I have a new profound respect for other’s cultures.” Along with their increased understanding of the historical roots of mathematics the students gained an appreciation of other cultures. All the students expressed a desire to travel abroad again, to continue to grow in their appreciation of the history and worldview perspectives of different cultures. This was as much of a goal for trip as the mathematical component. Only by understanding the cultural context can you truly appreciate the mathematics it produced.

8. Conclusion

Our experiences on this Math History Study Abroad Program confirmed that math history is best learned in a cultural context and that on site experiences add to understanding and appreciation of the achievements of mathematicians. A program like this has benefits for all math, computer science and science students. Future teachers would especially benefit. Actually seeing and experiencing the mathematical creations they had previously only read about, made mathematics “come alive” for the students. It imparts the view of mathematics as a continually developing human activity occurring within the framework of the surrounding culture. A Math History Study Abroad can change the way your students view mathematics.

Appendix I

Syllabus for History of Mathematics Study Program
Italy, Germany and England
January 2011

Course Concept:

In his essay on *The Study of the History of Mathematics*, George Sarton describes the history of mathematics as a "secret history." What he means is that, despite the importance of mathematics in western culture and especially in western science, most people know nothing about it. From the point of view of this vast majority, mathematics is an activity done in secret. "Yet that secret activity is fundamental; it is all the time creating new theories, which sooner or later will set new wheels moving, new machines working, or, better still, will enable us to obtain a deeper understanding of the mechanism of the universe."

The aim of this course is to gain an understanding of how mathematical ideas have developed over time, how social, cultural and historical factors have influenced the development of mathematics and conversely, how mathematics contributed to society and human culture. The course traces the historical development of mathematics from ancient to modern times with an eye toward situating bare mathematical facts into meaningful intellectual and historical context. Students will "meet" the mathematicians, read from their original writings, and see where they lived and what they created.

Relation to overall mathematics curriculum:

This course is designed to give historical background and cultural context for many of the ideas we teach in our undergraduate mathematics curriculum. It is designed to give life to the students' knowledge of mathematics; to provide an overview of mathematics so that the student can see how various courses fit together and to see where they come from. This course will help the students see that mathematics is a part of our culture. It will help future teachers of mathematics to see how they might use the history of mathematics in their teaching.

Course goals

Content goals:

- Follow the development of mathematics from early number systems to the current mathematical discoveries.
- Understand the relationship of mathematics to other disciplines such as science, engineering, architecture and art.
- Understand several historical developments in mathematics improve your ability to read mathematics texts.
- Improve your ability to read mathematics texts, some of them in their original writings.

Historical perspective goals:

- Study the mathematics of various different civilizations, their conception and use of mathematics, and how the historical conditions of those civilizations affected and were affected by mathematics.

- Develop your capacity to understand the contemporary world in the larger framework of tradition and history.
- Explore methodological and philosophical controversies.
- Focus on the problems of interpreting the past and can also deal with the relationship between past and present.
- Introduce the ways scholars think critically about the past, present and future.
- To impart the view of mathematics as a continually developing human activity.

Other goals:
- Improve your ability to read and think critically about mathematics by understanding the context in which it developed.
- Develop your ability to present mathematics and history in spoken and written forms
- Help you practice research skills including learning proper ways to use mathematics resource materials in the library and on the Internet.
- Satisfy, in part, your curiosity of how mathematics developed and how it fits into culture
- Explore some relationships between issues of the Christian faith and mathematics.

<u>Preparation</u>

Fall 2010 – 1 Credit Prep Class
We will meet weekly in the fall, working our way through two books: *Mathematics in Historical Context* and *Brunelleschi's Dome*. We will also gather for three evening meals (one Italian, one German and one English) as we watch three movies: *Galileo's Battle for the Heavens, Sir Isaac Newton: The Gravity of Genius, and Einstein's Big Idea*. *Mathematics in Historical Context* provides a cultural and historical context for the lives and work of mathematicians from Prehistory to Modern Times. It begins with an overview of the development of mathematics in non-European and ancient civilizations before. We will see examples of mathematics from these cultures in various museums (Pergamon in Berlin, British Museum in London). It then goes on to tell the story of how mathematics developed in Europe and America from the Middle Ages to the Modern Age. It tells the story of mathematics through biographies of people who did mathematics within a cultural setting of art, architecture, warfare, religion, navigation and medicine. We will encounter evidence of these people's impact and work as we travel. We will see the buildings they planned, the art they produced, the equipment they worked with, their inventions, and some of their original writings. . By reading Brunelleschi's Dome the students will have studied about the culture of Renaissance Italy and the way mathematics was integrated into the political, cultural and practical aspect of life. They will see evidence of what they have read as we travel through Italy. The three movies each provide a broad historical and cultural context for major discoveries made in early Renaissance period, the Scientific Revolution and the Nineteenth and Twentieth Centuries. They reinforce the development aspect of mathematics and science and its interrelationship with cultural and religion.
Students will research a mathematician (chosen from a given list) and write a five page biographical paper in anticipation of a presentation to the class during the trip. In the paper and the class presentation, the student will give an introduction to their mathematician, with details about their personal and professional life as well as placing them in a cultural context. They will explain how this mathematician impacted the culture of their time, explain the role he/she played

in the development of mathematics, and what evidence of their work we might see in the cities in which we travel.

Jan 2011 – Sites/Learning Objectives

Rome: Awareness of the interplay between Roman engineering and the spread of the Roman Empire. Romans borrowed from other cultures (like the Etruscans), making modifications. We will see evidence of the scope of their building projects both in terms of size and ingenuity, and learn how these were built (slave power) and financed (military expansion). Viewing Roman sites will help students appreciate the significance of Roman arches, their aqueduct water system, and architecture (dome structure of Pantheon). We'll see influence of Roman architecture and engineering in the other cities we visit, especially Florence and the sites in England.
Florence and Pisa: This was the birthplace of the Renaissance. Galileo, Leonardo DaVinci, Michelangelo and many other famous Renaissance figures lived and worked here. We'll get to know these important mathematicians/scientists/artists within their cultural setting by seeing where they lived and worshiped, the instruments and tools they and the works they produced. We'll see the role mathematics played in architecture, art, and inventions. We'll learn about the political make-up of Italian city-states of the Renaissance and the role that played in the development of mathematics.
Berlin and Goettingen: Euler and Gauss lived and worked in these cities for much of their lives. They were the center of European mathematics in the 19th and early 20th century. We'll learn about the mathematicians who lived and worked in these cities and the reasons behind the rivalries between these cities. We'll have tours of the Berlin Academy of Sciences including Eulerian Archives, with a talk on German mathematics by Dr. E. Knobloch, a mathematician with the Berlin Academy. We will visit Humboldt University and meet with mathematicians there. In Berlin we will visit museums containing archeological treasures from Babylonian, Roman, Grecian and Islamic cultures – reinforcing concepts from our study of ancient mathematics. A historian will take us on a guided walking tour of the city, pointing out important mathematical sites. In Gottingen, we will have a Gauss-specific tour followed by a visit to the Mathematical Institute and Gauss's observatory. In both cities we will learn how politics and religious persecution destroyed these centers of mathematics. We'll see how post-WWII and post-Berlin Wall Germany is responding to lessons of the past and rebuilding itself both as a city and a mathematical center.
Greater London Area: This was a center for the development of the Scientific Revolution. The British Royal Society promoted the new experimental method as opposed to accepted method of relying on writings of ancient scholars for truth. The head librarian of the British Royal Society will give us a tour of Royal Society archives (including many of Newton, Boyle, Hooke and Captain Cook's original documents). We'll visit the Science Museum and the British Museum which have exhibits of scientific instruments, history of math and computers, and exhibits of mathematical tables from ancient cultures. We'll see how mathematics intersected with science, commerce and navigation. We'll visit Christopher Wren's churches, seeing the mathematical influence within them. We'll travel to Greenwich to see the Royal Observatory and the Maritime Museum (mathematical influence on astronomy and navigation). The day trip to Oxford will include a guided tour of the History of Science Museum there (includes many 16th-17th century scientific instruments), a guided tour of Bodleian Library and a tour of the Sheldonian Theatre (Christopher Wren building). We'll take a Math/Science Walk around the city visiting key math

sites. Bletchley Park, another day trip from London, was the WWII cryptography center, where we'll have a guided tour showing us the role mathematics and computers played in WWII.

Course Requirements:
One-Credit Fall 2010 Prep Course – successful completion
Readings from Course Reading Packet; participation in group discussions on readings
10-15 Minute Biographical Presentation of an Italian/German/English mathematician (based on Prep Course biographical paper)
Full Attendance at all seminars, group discussion meetings and touring activities
Daily Journal entry adding culture experiences/perspective to Course Readings and Biographical Presentations
Final Reflective Essay on what they learned about the intersection of mathematics and culture giving concrete examples from each country we visited.

Assessment:
Biographical Presentation – 25%
Journal – 30%
Preparation and active participation in group discussions on readings: 30%
Final Reflective Essay 15%

Contents of Course Reading Packet

Reading Packet #1:
1. Liberal Arts/Medieval Mathematics Readings
   Who Invented the Algebra by Robin Wilson
   http://www.gresham.ac.uk/printtranscript.asp?EventID-378
2. Renaissance Perspective
   a. Alberti's *On Painting*
   http://www.noteaccess.com/Texts/Alberti/ Read the Introduction, Prologue and Book I
   b. Brunelleschi and the *Origins of Perspective*:
   http://www.dartmouth.edu/~matc/math5.geometry/unit11/unit11.html
3. Algebra
   a. Cardano and Bombelli: Fauvel's *The History of Mathematics ~ A Reader* p. 253-265
   b. Stevin and Viete: Fauvel's *The History of Mathematics ~ A Reader* p. 270-275

Reading Packet #2:
1. Scientific Revolution 1620-1720
   a. Overview: Calinger's *Classics of Mathematics* p. 291-321
   b. Kepler: Fauvel's *The History of Mathematics ~ A Reader* p. 320-323
   c. Galileo: Fauvel's *The History of Mathematics ~ A Reader* p. 328-335
2. Age of Enlightenment and the French Revolution 1720-1800
   a. Euler: Calinger's *Classics of Mathematics*, p. 486-489
      Fauvel's *The History of Mathematics ~ A Reader*, p. 447-454, p. 468-471
   b. *Mathematics at the Prussian Academy of Sciences 1700-1810* by Eberhard Knobloch

p. 1-8 in *Mathematics in Berlin* by Heinrich Begehr and Helmut Koch

3. 19th Century
   - a. Overview: Calinger's *Classics of Mathematics* p. 521-524
   - b. Gauss: Fauvel's *The History of Mathematics ~ A Reader* p. 485-498; Calinger's *Classics of Mathematics* p. 530-536 (FTA)
   - c. Cantor and Dedekind: Fauvel's *The History of Mathematics ~ A Reader* p. 572-581

Reading Packet #3:

1. 20th Century Mathematics
   - a. Overview: Calinger's *Classics of Mathematics* p. 681-684
   - b. Bertrand Russell: Calinger's *Classics of Mathematics* p. 668-679,
   - c. David Hilbert: Calinger's *Classics of Mathematics* p. 696-717
   - d. Brouwer: Calinger's *Classics of Mathematics* p. 732-734
   - e. Gödel: Calinger's *Classics of Mathematics* p. 744-745
   - f. Goettingen: *Mathematics at Gottingen Under the Nazis* by Saunders MacLane
     http://www.ams.org/notices/199510/maclane.pdf
     Episodes in the Berlin-Gottingen Rivalry, 1870-1930 by David E. Rowe,
     The Mathematical Intelligencer, Vol. 22, No. 1, 60-69, Springer-Verlag
   - g. *Mathematics in Berlin, 1810-1933* by David E. Rowe, p. 9-27 in *Mathematics in Berlin* by Heinrich Begehr and Helmut Koch
   - h. United States: *Refugee Mathematicians in the United States, 1933-1941; Reception and Reaction* by Nathan Reingold;
     http://www.ams.org/samplings/mathhistory/hmath1-reingold13.pdf

Reading Packet #4:

1. Foundations of Calculus
   - a. Calculus: *Touring the Calculus Gallery* by William Dunham:
     http://mathdl.maa.org/images/upload_library/22/Ford/dunham1.pdf
   - b. Leibniz: Calinger's *Classics of Mathematics* p. 383-394;
   - c. Newton: Calinger's *Classics of Mathematics* p. 395-398;
     Fauvel's *The History of Mathematics ~ A Reader* p. 381-383;
     Newton's Letters to Leibniz: Fauvel's *The History of Mathematics ~ A Reader* p. 402-407;
     Biographical sketch by John Maynard Keynes: *Newton, the Man*, Fauvel's *The History of Mathematics ~ A Reader* p. 421-422
   - d. Bernoulli's: Calinger's *Classics of Mathematics* p. 418-428
   - e. Overview of calculus in the 18th century: Calinger's *Classics of Mathematic, s* p. 429-459
   - f. Berkeley: Calinger's *Classics of Mathematics* p. 469-474

3. Rigorization of Calculus
   - a. Cauchy/Bolzano: Fauvel's *The History of Mathematics ~ A Reader* p 555-556, 563-572

Appendix II

Syllabus for Prep Course
Fall 2010

Textbooks: *Mathematics in Historical* Context by Jeff Suzuki and *Brunelleschi's Dome* by Ross King

Purpose and Format of the Course

The purpose of this course is to prepare you to participate in the Jan Term study seminar to take place in Europe in January 2011. The course will provide an introduction to the development of mathematics in a cultural and historical context. We will meet for one hour each week (except movie nights), working our way through two books: *Mathematics in Historical Context* and *Brunelleschi's Dome*. We will also gather for three evening meals (one Italian, one German and one English) as we watch three movies: *Galileo's Battle for the Heavens*, *Sir Isaac Newton: The Gravity of Genius*, and *Einstein's Big Idea*. *Mathematics in Historical Context* provides a cultural and historical context for the lives and work of mathematicians from Prehistory to Modern Times. It begins with an overview of the development of mathematics in non-European and ancient civilizations. We will see examples of mathematics from these cultures in several museums, including the Pergamon in Berlin and British Museum in London. Suzuki then goes on to tell the story of how mathematics developed in Europe and America from the Middle Ages to the Modern Age. It tells these stories through stories of people who did mathematics within a cultural setting of art, architecture, warfare, religion, navigation and medicine. We will encounter evidence of these people's impact and work as we travel. We will see the buildings they planned, the art they produced, the equipment they worked with, their inventions, and some of their original writings. By reading *Brunelleschi's Dome* we will learn about the culture of Renaissance Italy and the way mathematics was integrated into the political, cultural and practical aspect of life. Again, we will see evidence of what we have read as we travel through Italy. The three movies each provide a broad historical and cultural context for major discoveries made in early Renaissance period, the Scientific Revolution and the Nineteenth and Twentieth Centuries. They reinforce the development aspect of mathematics and science and its interrelationship with cultural and religion.

Assignments

1) Reading Responses/Reading Quiz

Reading Responses: For each week's reading there will be a list of Reading Questions for you to respond to in a reflective and insightful manner. These will be posted on the Discussion Board section of Blackboard. Write and post your response by Tuesdays at midnight. Read your classmates' responses and respond to two of them (different ones each week until you have responded to one from each of your classmates) before 5 p.m. on Wednesdays. There will also be one reading quiz on the book *Brunelleschi's Dome.*

2) Class discussions and activities

Be prepared to discuss the class readings and actively participate in class activities.

3) Biography of a Mathematician

Students will research a mathematician (chosen from a given list) and write a 7-10 page biographical paper in anticipation of an oral presentation to the class during the Jan 2011 trip. In the paper and the class presentation, you will give an introduction to your mathematician, with details about their personal and professional life as well as placing them in a cultural context. You will explain how this mathematician impacted the culture of their time, explain the role he/she played in the development of mathematics, and what evidence of their work we might see in the cities in which we travel.

List of Mathematicians for Paper and Presentation
(The city for the oral presentation is listed in parenthesis.)

Group 1 Presentations (Italy):
Luca Pacioli 1445-1517 (Rome)
Albrecht Durer 1471-1528 Gerardus Mercator 1512-1594 (Florence)
Tycho Brahe 1546-1601 (Florence)
Girard Desargues 1591-1661 (Rome)
Maria Gaetana Agnesi (1718-1799) (Rome)

Group 2 Presentations (Germany):
August Leopold Crelle 1780-1855 (Berlin)
Bernard Riemann 1825-1866 (Goettingen)
Daniel Bernoulli 1700-1782 (Berlin)
Issai Schur 1875-1941 (Berlin)
Emily Noether 1882-1935 (Goettingen)
John von Neumann 1903-1957 (London)

Group 3 Presentations (England):
John Wilkins 1614-1672 (Oxford)
Christiaan Huygens 1629-1695 (London)
Isaac Barrow 1630-1677 (Oxford)
Edmund Halley 1656-1742 (Greenwich)
Ada Lovelace 1815-1852 (London)
Alan Turing 1912-1954 (Bletchley Park)

Math History Prep Course Tentative Schedule

Thursday, Sept. 9 – Convocation = no class; read *Brunelleschi's Dome, Chapters 1-10*

Thursday, Sept. 16 – Movie: *Engineering an Empire – Rome.* Finish reading *Brunelleschi's Dome*.

Thursday, Sept. 23 – Chapter 5: The Middle Ages; Movie: James Burke *Connections*

Sunday, Sept. 26 at 5:30 p.m. Italian dinner and movie on Galileo

Thursday, Sept. 30 - Chapter 6: Renaissance and Reformation
Reading quiz on Brunelleschi's Dome; Movie: *Engineering an Empire – DaVinci's World*;

Thursday, Oct. 7 - Chapter 6: Renaissance and Reformation continued; Engineering an Empire: Da Vinci's World continued (Art of War)

Thursday, Oct. 14 - Babylonian/Islamic World/Ancient World; Movie *Story of 1*
Read and work through the following information on Babylonian Mathematics (source: The Open University: http://openlearn.open.ac.uk/mod/oucontent/view.php?id=397245&direct=1)
In class: British Museum Rhind Papyrus (or Movie *Story of 1*)

Thursday, Oct. 21- Chapter 7: Early Modern Europe – Lecture on the British Royal Society

Thursday, Oct. 28 - Chapter 8 The Eighteenth Century; Movie: *Longitude*

Thursday, Nov. 4 - Chapter 9: The Nineteenth Century; Class discussion on reading

Thursday, Nov. 11 - Chapter 10: The United States: Founding of AMS/MAA

Thursday, Nov. 1 - Chapter 11: The Modern World; Rough draft of paper due; Bletchley Park

Sunday, Nov. 21 at 5:30 German dinner and movie on Einstein

Thursday, Dec. 2 Journal Articles: *Episodes in the Berlin-Goettingen Rivalry 1870-1920* by Jeremy Gray and *Refugee Mathematicians in the U.S.A. 1933-1941: Reception and Reaction* by Nathan Reingold

Thursday, Dec. 9 – Travel tips

Appendix III

Math History Study Program Sample 3-Week Itinerary
(Italy-Germany-England)

Day 1
Fly to Rome
Day 2 Rome
Arrive in Rome, tour ruins of Ostia Antica
Day 3 Rome - Renaissance Art and Architecture
Vatican Museum, St. Peter's Basilica
Night Walk across Rome: Campo de' Fiori, Piazza Navona, Pantheon, Piazza Colonna, Trevi Fountain, Spanish Steps
Day 4 Rome – Ancient Rome, Engineering Wonders
Coliseum, Forum, Palatine Hills, Mamertine Prison, Trajan's Column, Campidoglio Michelangelo's Plaza), Victor Emmanuel Monument, Pantheon
Day 5 Rome Free Day
Explore Rome -Suggested itineraries: Villa de'Este in Tivoli; Catacombs and Appian Way; Baths of Diocletian
Day 6 Rome->Florence – Renaissance Engineering and Art
Morning Train to Florence
Orientation to Florence: City/Walk – Duomo, Baptistery Doors/Gates of Paradise, Giotto's Tower, Palazzo Vecchio, Uffizi Courtyard, Ponte Vecchio, Orsanmichele Church
Uffizi Gallery (use of math and science in art)
Accademia (Michelangelo's *David*);
Santa Maria Novella (perspective drawing/architecture)
Climbing the Dome (see Brunelleschi's construction techniques + great view of Florence)
Day 7 Florence – Galileo, Leonardo da Vinci and Michelangelo
Duomo Museum (featuring Brunelleschi's dome, Ghiberti's bronze doors, Donatello's and Michelangelo's statues)
Science Museum (Galileo's instruments) - didactic tour
Santa Croce Church (Michelangelo's and Galileo's grave)
Leonardo Museum (working models of DaVinci's inventions)
San Lorenzo Markets (shopping)
Day 8 Florence Free Day
Suggested Itineraries: Siena/Fiesole/Explore Florence
Day 9 Florence/Pisa - Galileo
Day trip to Pisa: Galileo sites, Leaning Tower of Pisa, Duomo
Overnight Train from Florence to Berlin
Day 10 Berlin – 19th and 20th Century German History
Orientation in Berlin, German history, Humboldt University, Unter Der Linden area (Brandenburg Gate)
Guide: German mathematicians in a historical context
Day 11 Berlin Antiquity/Judaism/Math Tour of Berlin

Museum Island, Pergamon Museum (antiquity, Pergamon Altar, Market Gate from Milet, Islamic Art, Babylonian and Sumerian architecture and collections)
Jewish Museum
Math Tour of Berlin
Day 12 Berlin – Euler Archives
Berlin Academy/Euler Archives (Talks on Gauss, Euler and Cantor and guided tour of Euler Archives
Meal with German Christian Students
Tour of Turkish Neighborhoods and West Berlin
Day 13 Goettingen – Gauss, Riemann and 19th Century Mathematics
Day trip to Goettingen: Science/Math Guided Tour of Goettingen including Gauss' Observatory and the Mathematical Institute
Day 14 Berlin->London
Morning flight from Berlin to London
Orientation to London: Visit British Library, Trafalgar Square, Buckingham Palace, Parliament Building, Big Ben, Downing Street
Evening service at Holy Trinity Brompton
Day 15 London
St. Paul's Cathedral tour (Christopher Wren's masterpiece – an architectural and mathematical wonder)
British Royal Society tour (18th Newton's papers, papers from Scientific Revolution)
Science Museum – math and computers history floor
Day 16 Oxford
Guided Tour of Oxford History of Science Museum (workshop on astrolabe);
Tour of Bodleian Library and Sheldonian Theatre (Wren's engineering)
Lunch at Eagle and Child
Tour of Christ Church College followed by Evensong
Day 17 Greenwich
City Cruise boat ride/tour to Greenwich
Greenwich Royal Observatory and Maritime Museum
Evensong at Westminster Abbey – 5 p.m.
Day 18 Bletchley Park
Guided tour of Bletchley Park – WWII Cryptography Center
Late Afternoon: British Museum- 18th century science floor
Day 19 Bath or Cambridge
Bath: Roman Baths
Cambridge: Newton's college; Wren library
Day 20 London Free Day
Free Day
Day 21
Fly home

Appendix IV

Organizational Timeline
For Planning a Study Abroad Program

1. Two to Three Years in Advance of Your Travel

Know Your Parameters
Is this a school sponsored travel program? What school parameters must you consider in terms of length of time, budget, course approval, students going, travel planning assistance, etc.?

Working with those parameters, decide on a reasonable travel plan. Studying math history is like studying the history of civilization – it's a huge topic so you'll need to limit yourself geographically and with respect to the time period. Think about your goals for this travel abroad program. Do you want to give your students snapshots from the history of math or an in-depth examination of a particular period/topic? What do you want your students to take away from this experience?

Background Research
Read books and articles that deals specifically with places and time periods that will be a focus of your trip. In addition to math history books read biographies, historical novels, architecture and art books to get a sense of the cultural milieu in which mathematics was done. You don't need to become an expert in these areas, but it will help you to determine what you want your students to see and learn, and how you want them to acquire that knowledge. You may decide to tap into local guides/experts for specific topics.

If at all possible do a preliminary scouting trip, traveling to the places to which you plan on taking your students. Some universities have exploratory travel funds for new study abroad programs. Travel as if your were on a study abroad program, i.e. stay in the same type of hotels/hostels, use the same type of transportation, and set the same pace of site-seeing you expect of your students. This will help to give you a realistic picture of what kind of itinerary you want to plan for your program. As you travel explore museums and science societies. Try to make local contacts with people there, asking them about workshops and educational talks they offer for visitors. Try to get a sense of how much time you want students to spend at each site or in each museum – this will be important as you plan out your itinerary.
Keep a journal as you travel with notes – contacts, noteworthy things you saw, travel notes including transportation, local customs regarding restrooms, water, ordering food, meal times, tipping.

2. One to Two Years Advance of Your Travel

Planning Your Itinerary and Budget
Having determined the focus of your math history study abroad program you want to determine which sites and experiences you want to incorporate into your trip that will enhance student

learning. If you were able to do a scouting trip you may have made a list of things you want to do and see. Taking these items, and incorporating them into an itinerary, can be a logistical challenge. Learn from others who have traveled. Faculty colleagues who have led study abroad programs have been generous in sharing travel advice, hotel and restaurant recommendations, curriculum ideas, and budget guidelines. History sessions at math conferences bring together people who often are good sources of information about must-see mathematical sites/items. For example, at a math history workshop at a Joint AMS/MAA meeting, I learned that you could request special permission to see the Rhind Papyrus at the British Museum. The papyrus is too delicate to put on display in the main area of the museum, but an email request will allow you to make an appointment to have a special guided tour of the papyrus and related artifacts. Another source of information is the MAA Study Tours (see the MAA website for more information). While most of these study tours focused on a single country, you can learn some of the mathematical highlights of that country by reading over the itinerary for that particular study tour.

Once you have an idea of which cities and places you want to see, research dates and costs. Know the opening and closing times and days for museums, transportation timetables, holidays and seasonal costs. I used Rick Steves travel books as a beginning for this exploration, and followed up by checking the websites of corresponding places I wished to visit. Some examples: While January is usually considered off-season with cheaper hotel rates and shorter lines, exceptions to this rule exist. The first week of January, until the Feast of the Epiphany on January 6th, is a holiday week in much of Italy and so lines for museums can be much longer; I recommend paying extra for reservations. The second week of January is “Fashion Week” in Florence and hotel rates triple. Many stores, businesses and sites close on Sundays in Germany.

Put together a reasonable travel itinerary, figuring in costs for each museum/site, costs for local guides, transportation, hotels and food costs and keeping in mind open times and days may vary. Note that each move to a different city or hotel requires additional arrangements, travel time and transportation costs. Student exhaustion amplifies with each logistical move. Try to balance time spent in museums or lectures with open-air exploration, and especially free/down time. Give students time to explore on their own. Build in time for laundry, emailing and shopping. Build in time for group discussion/processing/reflection.

Curriculum

Decide on what and how you want your students to learn. Some of this can be accomplished in advance in a Prep Course offered the semester before traveling. But as you travel, do you want to be giving regular lectures? When it is appropriate to utilize local guides? Trade-offs exist for each. Local guides, especially trained math guides or museum educational guides with expertise in a particular area can provide a richer educational experience for some sites. However, you lose some flexibility in your schedule when you use guides, as these require set appointments. If you do plan on doing some lecturing as you travel you need to consider where you will conduct your lectures, especially if you travel during the winter months when it is too cold to sit outside. Many hotels in Europe have breakfast rooms and some hotels will allow you to use these rooms for an afternoon or evening lecture.

Journaling and site-reports provide good ways to help students process and reflect upon what they experience. Develop some specific questions and guidelines for these reflections, but also give the students a chance to add their own remembrances. Some teachers use exams for assessment; others have students do presentations. Again, talk to other faculty who have led study abroad programs to hear the pros and cons of each type of assessment to ascertain which methods will work best for you.

Build in Memorable Experiences
Build in opportunities to experience local culture. Take advantages of plays in London, local markets in Florence, a boat ride through the center of the city or hiking in countryside. Try to set up meetings with local students or businesses in the countries you visit. Churches often have free concerts in the evening – a great way to experience architecture and local music at a low cost! Take your students to a variety of churches – mass at a Catholic Cathedral, evensong at Anglican church, a Sunday service at an evangelical church – to give them a feel for the variety of Christian expressions of worship. Have some group meals, experiencing local cuisine together.

3. The Year Before You Travel

Advertising, Budget, and Student Preparation
Advertise your course. Put together a slide show or a poster, have an information sheet with tentative itinerary and costs. Screen the students who apply to your program. Traveling abroad, with a set pace and itinerary, shared sleeping quarters, unfamiliar food and settings, can be stressful. Talk to the students about their expectations and goals for the trip.
Prepare your budget. Check and recheck local prices and transportation costs. Build in extra money for the unexpected. Finalize contacts with local guides.
Work with your school's Study Abroad Office to purchase tickets (transportation, local concerts) and pay travel guides.
Help your students prepare for the study abroad experience. You may want to consider a prep course in which you cover math history curriculum as well as travel and safety guidelines. Building a sense of camaraderie among the students in advance often makes travel less stressful and more of an adventure to be shared.

4. Additional Suggestions and Lessons Learned

Travel Safety
Use a buddy system for accountability and safety in travels.
When you reach a new locale make sure each student has a map of the area and a business card from your hotel/hostel. Both of these are usually available at the front desk. The students can use the map to navigate their way around in the city. If the students get lost, and especially if they don't speak the language to ask for directions, they can show the business card to a taxi driver and arrive back at the hotel.

Careful Preparation Pays Off!
Research on curriculum, travel sites, establishing contact with local guides/experts, logistical arrangements, etc. can help make the trip go very smoothly. There will always be unforeseen challenges, but you can minimize the foreseen ones.

Choose your students well.
Having responsible students who looked out for each other made traveling with them a joy. I was able to give the students a lot of responsibility as we traveled – let them be the travel guides in places – which gave them confidence in their travel skills as well as added to their appreciation of the places we visited.

Blogging
Blogging and journaling gave the students a chance to reflect and to tell their stories. These were also a good record of what transpired that could be used for future planning.

Relax and Enjoy Your Time With the Students
Be flexible – not everything has to go smoothly/perfectly for this to be a wonderful experience for all. In fact, overcoming some adversity teaches students how to adapt and be prepared for future travel/life. Often life long friendships result from this shared experience.

Take notes for the future
Keep a journal with notes of what went well, challenges you faced, and changes you want to make for the next trip. If you made new contacts during your travels, keep their business cards in your journal for future reference. Keep in mind that this course is a work in progress.

Schedule a reunion within a month or two of your return
Students will enjoy the chance to reconnect and share remembrances and photos. Students have also had a chance to process what they saw and learned and good discussions can ensue as you talk about cultural differences, and their new perspective on math and culture. This is also a good time to evaluate the program together and discuss changes for future trips.