# Teaching Math Through Stories

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Stories have the potential to impart both knowledge and wisdom. From them we can develop empathy, master facts and concepts, and visit distant places and times - including imaginary worlds. Powerful stories may be short (such as the parables of Jesus or the fables of Aesop), or quite long (such as some novels by Tolstoy, Dostoyevsky, and Tolkien). Mathematics has not traditionally been taught using stories (if we ignore the "Johnny has 3 apples and Betty has 7" variety). But some quite good resources *are* available that effectively present mathematical concepts.

The goal of this review is not to present a scholarly treatise on stories and mathematics, nor to present a discussion of how mathematical stories relate to faith. Instead, the purpose is to introduce a few worthy examples that I have encountered, with the hope that you might find some of them interesting and useful. The examples all seek to interest the reader in some mathematical ideas that do not typically appear in the standard curriculum, but should nevertheless be accessible to an interested reader. Of course, the background preparation for "an interested reader" will vary a bit – from mid high school to mid college with a math/science background.

Since most readers of this journal have an interest in expanding the ranks of those who take joy in beautiful mathematical ideas and clear exposition, I present for your consideration these books that succeed admirably in that endeavor.

## The Man Who Counted
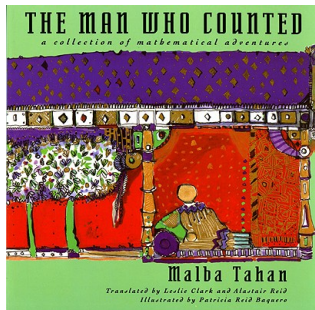


*The man who counted: a collection of mathematical adventures* is supposedly by Malba Tahan but was really written by the Brazilain mathematician Júlio César de Mello e Souza in 1938. A good English translation has been published by W. W. Norton, ISBN 978-0-393-30934-8.

The book follows the rise to fame of Beremiz Samir, a fictional citizen of the Abbasid Caliphate. The time period is the first half of the 13th century - the final years of the Islamic Golden Age. The fictional author (Malba) encounters Beremiz in Samarra and they travel together to Bagdad - the center of power and culture and learning. This is an ideal setting for the story as scholarship was highly valued: even Jewish and Christian scholars were welcome at the court of the Caliph. This is also the environment in which the Persian mathematician Muhammad ibn Mūsā al-Khwārizmī published his algebra textbook. "The Abbassids were influenced by the Qur'anic injunctions and hadith such as 'the ink of a scholar is more holy than the blood of a martyr' stressing the value of knowledge."[1]

The book consists of 34 short chapters, most containing a single mathematical puzzle that Beremiz must solve. The puzzles are interesting and usually capable of being solved by people with a good high school education. I have successfully used the book in a junior-senior level combinatorics course. The only downside is that the solutions are usually presented right after the puzzle, so it takes some self-discipline to stop reading and work on the puzzle yourself.

Here is a typical puzzle (from chapter 23).

> A rajah on his death left to his daughters a certain number of pearls with instructions that they be divided up in the following way: his eldest daughter was to have one pearl and a seventh of those that were left. His second daughter was to have 2 pearls and a seventh of those that were left. His third daughter was to have three pearls and a seventh of those that were left. And so on. The youngest daughters went before the judge complaining that this complicated system was extremely unfair. The judge, who, as tradition has it, was skilled in solving problems, replied at once that the claimants were mistaken, that the proposed division was just, and that each of the daughters would receive the same number of pearls.
>
> How many pearls were there? How many daughters had the rajah?

## The Housekeeper And The Professor
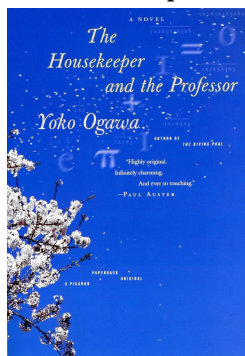
*The housekeeper and the professor* is a novel by prolific Japanese author Yoko Ogawa. An English translation has been published by Picador, ISBN 978-0-312-42780-1.

 *The man who counted* consists of a series of short episodes in the life of Beremiz. There is plot development, but the focus is definitely on the puzzles. As such, there is no real character development. *The housekeeper and the professor* is a full-blown novel, with both plot and character development. The mathematics and the professor are most likely inspired by the biography of Paul Erdős.[2]

 The novel focuses on three main characters: the professor (a mathematician who has suffered some brain damage in an accident and now has only an 80 minute window for short-term memory), the housekeeper (a single mom with a 10 year-old son and not much formal education), and the son (nicknamed "root", after $\sqrt{\ }$, by the professor).

The novel traces the growing respect and affection between these three characters as the housekeeper gradually comes to understand the professor's love of mathematics and starts making some attempts on her own to learn mathematics and solve some puzzles given to her by the professor. The professor and root share a common love of baseball, complicated by the professor's mistaken belief that the teams and players are still the same as before his accident.

The mathematics does not dominate the novel as it does in *The man who counted* and the following book, *Math girls*. However, there is a substantial amount of significant math (mostly from number theory) present in the book. The exposition of the mathematics is clear and well-motivated. The book has won at least one literary award in Japan and was a best-seller there. The book is rich in dramatic content which is effectively used to elaborate the personalities and histories of the two main characters.

Here is a brief extract from chapter 7. The housekeeper mentions elsewhere that the professor had taught her the sieve of Eratosthenes, but she had forgotten the details.

> *Taking my cue from the Professor, I started carrying a pencil and a notepad around in the pocket of my apron. That way, I could do my calculations whenever the mood struck. One day while I was cleaning in the kitchen of the tax consultants' house, I found a serial number engraved on the back of the refrigerator door: 2311. It looked intriguing, so I took out my notepad, moved aside the detergent and the rags, and set to work. I tried 3, then 7, and then 11. All to no avail. They all left a remainder of 1. Next I tried 13, and 17, and 19; but none of them was a divisor. There was no way to break up 2,311; but, more than that, its indivisibility was positively devious. Every time I thought I had spotted a divisor, the number seemed to slip away, leaving me oddly exhausted yet all the more eager for the hunt — which was always the way with primes.*
>
> *Once I'd proved that 2,311 was a prime, I put the notepad back in my pocket and went back to my cleaning, though now with a new affection for this refrigerator, which had a prime serial number. It suddenly seemed so noble, divisible only by one and itself.*

## Math Girls
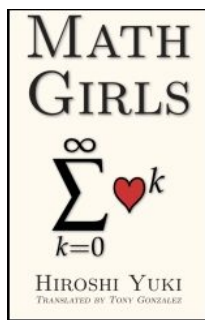
*Math girls* is a young adult novel by Hiroshi Yuki, a Japanese computer scientist with an extensive list of books written for that field. An English translation of *Math girls* has been published by Bento Books, ISBN 978-0-9839513-0-8.

 The mathematics takes front seat in this novel. There is a thin story to keep things moving. That story centers on an unnamed math-loving male high school student and his relationship with two young women who attend his school. One, Miruka, is even brighter than the narrator – continually offering him challenges to solve. The other is a less gifted younger student, Tetra, who asks him to teach her math (as a way to spend time with him). There is some mild romantic tension until Miruka comes to accept Tetra's genuine interest in math and also her willingness to work independently on problems.

The mathematics is varied and often non-trivial. Topics include infinite series, generating functions, falling factorials, Catalan numbers, and many others. Wikipedia has a list of the topics mentioned in the book.[3]

The exposition is clear and interesting, taking inspiration from *Concrete Mathematics* by Graham, Knuth, and Patashnik. If the front cover has not enticed you to read this book, then hopefully the full list of topics will inspire you to find a copy of the book. It is quite suitable for undergraduates and bright high school students who like math.

Here is the beginning of chapter 8.

> "Hey!" Tetra came running up to me after school, in front of the school gates. "There you are! I didn't see you in the library, so I wondered what was wrong. You on your way home? Would it be okay if I – hey, what's that?"
>
> I handed her the card I was holding.

$$\boxed{\begin{array}{c} \textit{My Card} \\[6pt] H_\infty = \sum_{k=1}^{\infty} \frac{1}{k} \end{array}}$$

> "It's a challenge. The starting point for a hunt."
>
> "Huh?" Tetra looked puzzled.
>
> "Think of it as a …a forest with a hidden treasure. You know it's in there, the trick is finding it. Mr. Muraki gives us problems like this all the time."

This begins a chapter that winds its way through partial sums and limits, an exposition of propositions, a proof that $H_{n+1} - H_n > H_{n+2} - H_{n+1}$ for $n$ a positive integer, and a proof that the partial sums are unbounded. But the chapter is not finished. Miruka enters and we are presented with the zeta function and the connections between the continuous exponential function and the discrete falling factorial function. This naturally leads to the continuous logarithm function (the inverse of the exponential function) and its discrete analogue, which turns out to be a partial sum of the Harmonic series. Then Miruka introduces proof by contradiction and goes on to discuss Euler's linking of the Harmonic numbers to the infinitude of primes.

While this is not a book for those who instinctively shy away from math (the previous book is a better choice for them), it is an exiting journey for those who like mathematics.

**Manga Guides**
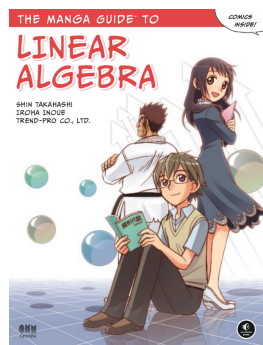


*The manga guide to linear algebra* by Shin Takahashi is one of a series of manga guides to various topics in mathematics, computer science, and science. English translations are being published by No Starch Press. This particular guide has ISBN 978-1-59327-413-9.[4]

Manga guides uses a mix of comic art and prose text to introduce their subject matter. There is always a story that keeps the topics flowing. In *The manga guide to linear algebra*, university mathematics student Reiji wants to learn karate. The karate club captain agrees to let him join the club if Reiji will tutor the captain's younger sister, Misa, in linear algebra. You can download chapter 2 and the appendices for free at http://nostarch.com/linearalgebra.

*The manga guide to linear algebra* covers some background material (logic, sets, functions, permutations and combinations), then moves on to matrices, vectors, a bit of vector space theory, linear transformations, and eigenvalues-eigenvectors. The presentation is clear and well-motivated. There are a few well-chosen examples and a small handful of exercises.

These guides are not replacements for a good textbook. Instead, they make nice supplemental material for the intended courses. I think they are effective if used as either a quick overview of the key topics in the course or as a handy review just before an exam.[5]

There are manga guides to statistics (non-calculus, service course level), and calculus available. Manga guides to regression analysis and to differential equations are in the works. I would certainly recommend the guides to linear algebra, databases, and statistics. I was not as impressed with the exposition in the guide to calculus.

# Notes

[1]http://en.wikipedia.org/wiki/Abbasid#Islamic_Golden_Age

[2]http://en.wikipedia.org/wiki/The_Housekeeper_and_the_Professor

[3]http://en.wikipedia.org/wiki/Math_Girls#Mathematical_topics_that_appear_in_the_book

[4]Some truth in advertising: I did the technical review of the English translation for this work. This consisted of making sure the terminology and notation were consistent with American conventions. There were also several places I was asked to write a few sentences to clarify an idea. However, the book is essentially the book that was published in Japan.

[5]I received a positive report from one student who used the *Manga Guide to Databases* as a study aid. He was convinced that as a result, he scored higher on the exam than he normally does.