An Augustinian Perspective on the Philosophy of Mathematics

James Bradley
Dept of Mathematics and Statistics
Calvin College
braj@calvin.edu

Introduction

Enlightenment thinkers saw the universe as mechanistic and mathematics as the language in which the universe is written. They viewed mathematics as eternal, as transcending human minds, and as comprehensible by human beings. Thus mathematics, from their perspective, is our best tool for understanding the secrets of nature. This outlook was nicely summarized by Morris Kline: (Kline, p.107)

> *In brief, the whole world is the totality of mathematically expressible motions of objects in space and time, and the entire universe is a great, harmonious, and mathematically designed machine.*

From a Christian perspective, however, the Enlightenment outlook is flawed. It privileges mathematics and science and dismisses other sources of knowledge such as intuition and divine revelation. It identifies reason with mathematical thinking and empiricism, thus devaluing reflections on ethics, values, justice, and origins. Although Descartes professed religious belief, subsequent Enlightenment thinkers tended to view human intellectual capabilities as sufficient for ordering society and providing for peace and prosperity. Thus these thinkers privatized religion and viewed it as an inappropriate topic of discussion in the public sphere. In short, Enlightenment thought replaced God with a particular form of human reason - mathematics and science.

The post-modern perspective is much less optimistic about mathematics. It tends to see mathematical knowledge not as originating in the underlying structures and principles of nature but in human thought, especially in natural language, its patterns, and its grammatical rules. Mathematics is seen more as a social agreement and, as such, it has some usefulness, but it's not seen as transcendent. For example, in a recent paper, Paul Ernest wrote, (Ernest)

> *To account for the apparent certainty and objectivity of mathematical knowledge I claim first that mathematics rests on natural language, and that mathematical symbolism is a refinement and an extension of written language. The rules of logic and consistency which permeate the use of natural language provide the bedrock on which the objectivity of mathematics rests. Mathematical truths arise from the definitional truths of natural language, acquired by social inter-action…the truths of mathematics are defined by implicit social agreement—shared patterns of behavior—on what constitute acceptable mathematical concepts, relationships between them, and methods of deriving new truths from old. Mathematical certainty rests on socially accepted rules of discourse embedded in our 'forms of life.'*

Postmodernism explicitly rejects modernism's claim of privilege for empiricism and mathematics and is far more receptive to other ways of knowing. But it tends to see mathematics purely as a human construct and to deny mathematics' transcendence. However, most mathematicians see mathematics as having an independent identity, apart from their own minds. (Davis, p.321-2)

So from a Christian perspective, neither approach seems satisfactory. Thus this paper presents an alternative, one articulated by Augustine of Hippo, c. 400 A.D., long before the Enlightenment, and argues that it is still a viable perspective today.

It first presents Augustine's views on the four classical questions of the philosophy of mathematics:

What is the nature of mathematical objects? (The ontological question)
How do we obtain knowledge of them? (The epistemological question)
What is the meaning of "truth" in mathematics? How do we account for the certitude of mathematical truths? (The truth question)
How do we account for the effectiveness of mathematics in describing the physical universe? (The effectiveness question)

It then addresses Augustine's concept of reason. Next it discusses what has happened to Augustine's perspective in the roughly 1600 years since he articulated it. It then addresses some problems with both the Augustinian perspective and the more recent secular perspectives. And it concludes with reflections on some implications of an Augustinian perspective on mathematics for the practice of mathematics by Christian mathematicians and their students.

Augustine's views on the four basic questions

Augustine wrote around ninety books plus many sermons and letters. He addresses mathematics in several places but his most detailed comments are in Book 2 of *On Free Choice of the Will* (*De Libero Arbitrio*).[1] So my comments in this section will focus primarily on that work.

Before we can look at Augustine's treatment of the basic questions, we need to look at his use of a few key words. Augustine was, of course, unaware of abstract mathematical structures like groups, rings, and vector spaces. But he was familiar with geometry and number theory. His discussion in *De Libero Arbitrio* centers on number. However, the Latin word, *numerus*, translated as "number," also occasionally has the sense of "pattern," "form," or "structure." Augustine uses *numerus* in this sense as well. Thus I am going to take "number" as conveying an intuitive sense of the abstract structures that mathematicians have formalized in recent years.

Another key word is "truth." Augustine distinguishes "truths" and "Truth." A truth is a necessary and therefore immutable proposition. Such truths are eternal and common to all minds that contemplate them. Augustine gives some examples of truths (Williams, pp.50-51):

One ought to live justly.

Inferior things should be subjected to superior things.

Like should be compared with like.

Everyone should be given what is rightly his.

The uncorrupted is better than the corrupt, the eternal than the temporal, the invulnerable than the vulnerable.

A life that cannot be swayed by any adversity from its fixed and upright resolve is better than one that is easily weakened and overthrown by transitory misfortunes.

Mathematical statements such as "7 + 3 = 10" are also examples of truths.

"Truth" comes close to being identified with God. Augustine says, (p.57), "This is our freedom, when we are subject to the truth; and the truth is God himself, who frees us from death, that is, from the state of sin." He describes Truth as a kind of light, possessed by all who perceive the same truths at a given moment but not changed by any of them. It transcends our minds; in fact it rules them and is therefore independent of them. It cannot be gained from sensible objects. Nevertheless, his use of "Truth" is somewhat vague and he doesn't clarify it beyond these comments.

"Wisdom" is also a key word. At one point, Augustine identifies wisdom with Truth, writing "For no one is happy without the highest good, which is discerned and acquired in the truth that we call 'wisdom.'" At another point, he identifies wisdom with Christ. *De Libero Arbitrio* is written as a conversation between Augustine and his close friend, Euvodius. In response to Euvodius' question, "I would very much like to know whether wisdom and number are both included in one single class. For as you have pointed out, wisdom and number are associated with each other even in Holy Scripture."[2] Augustine replies with a third meaning for wisdom (p.52):

So, given the fact that both wisdom and number are contained in that most hidden and certain truth, and that Scripture bears witness that the two are joined together, I very much wonder why most people consider wisdom valuable but have little respect for number. They are of course one and the same thing.

The ambiguity of his terminology and the difficulties of translation make it hard to be completely clear on how Augustine sees the relationship between God and mathematics. But I think it's safe to say a few things, namely that Augustine sees truths as being eternal and immutable propositions rooted in God's nature. Truths include (at least) the basic patterns according to which the world has been created; wisdom is knowledge of these truths and is closely identified with Christ.

Thus we can see Augustine's answer to the ontological question – he views numbers as ideas in the mind of God that have been there from eternity and that God has used in creating the physical universe. He says to Euvodius (pp.52-53),

So, just as there are true and unchangeable rules of numbers, whose order and truth you said are present unchangeably, and in common to everyone who sees them, there are also true and unchangeable rules of wisdom.

But wisdom gave numbers to everything, even to the lowliest and most far-flung things.

But when we begin to look above ourselves again, we find that numbers transcend our minds and remain fixed in the truth itself.

Etienne Gilson nicely summarizes Augustine's concept of divine ideas (Gilson, p.80).

In any case, the ideas are the archetypes of every species or of every individual created by God. Everything was created in conformity with a certain model, and the type to which man belongs is obviously not the same as that of the horse. Thus everything was created according to it proper model, and since everything was created by God, the models of things, or ideas, cannot subsist anywhere but in the mind of God.

On the epistemological question, Augustine regards mathematical knowledge as *a priori* knowledge. He argues that our understanding of "one" cannot be empirical because we never experience unity in this world – even if we encounter a single object, it can again be broken down into a multitude of parts. Thus "oneness" must be an idea that we bring to our experiences and that informs them. Similarly, he argues that mathematical knowledge cannot originate in our bodily senses because we all have an intuitive understanding of infinity but we never experience it. For both of these reasons, mathematical knowledge is more foundational than knowledge we acquire through our senses. He asserts that the elementary truths of mathematics are neither induced nor deduced but are present to all who think. For example, after giving a simple argument from number theory, he writes (p. 46),

For those enquirers to whom God has given the ability, whose judgment is not clouded by stubbornness, these and many other arguments suffice to show that the order and truth of numbers has nothing to do with the senses of the body, but that it does exist, complete and immutable, and can be seen in common by everyone who uses reason.

In contemporary jargon, Augustine is saying that the capacity to understand some truths such as the basic truths of mathematics was hard-wired into our brains as part of our creation. This provides an explanation of how we, as physical beings, are able to access truths that are not physical.

We can see Augustine's perspective on the truth question in our earlier discussion of terminology: mathematical truths are eternal, transcendent propositions independent of human minds; their certitude comes from the fact that they originate in God's thoughts.

Mathematics is effective in helping us understand the physical universe for two reasons: (1) God used the patterns expressed by mathematics in creating the universe, and (2) God built the capacity to do mathematics into the human mind. For example, Augustine writes, *So if you see anything at all that has measure, number, and order, do not hesitate to attribute it to God as craftsman. If you take away all measure, number, and order, there is absolutely nothing left.* (p.69)

…whatever pleases you in material objects and entices you through the bodily senses has number. Thus you will ask where that number comes from; returning within yourself, you will understand that you could neither approve nor disapprove of anything you perceive through the bodily senses unless you had within yourself certain laws of beauty to which you refer. (p.60)

Therefore, there can be no thing, however great or small, that is not from God. (p.64)

Thus, from Augustine's perspective, our thought is capable of grasping at least some of the structures of creation and those structures exist in the eternal mind of God.

Augustine's views on reason

Augustine's perspective on the nature of reason is quite different from the Enlightenment perspective. I will use it below in analyzing the secularization of mathematics that occurred after about 1850. In *De Libero Arbitrio*, Augustine writes "For we know only what we grasp by reason." He goes on to say that reason forms categories and definitions and organizes them. Gilson (p.270) offers an Augustinian definition of reason, that "Reason (*ratio*) is the movement whereby the mind (*mens*) passes from one of its knowledges to another to associate or disassociate them."

A famous Augustinian adage is "Faith seeking understanding." That is, faith comes first, understanding later. But faith does not precede reason – in fact, as Gilson expresses it (p.24),

…the very possibility of faith depends on reason…Reason, then, is naturally present before understanding, and before faith as well. If we were to belittle or hate reason, we should despise God's image within us and the very source of our preeminence over all other living creatures.

This seems to set reason above faith, but in fact it does not. Gilson continues (p.33),

No one doubts that man can know the truth, say of mathematics, without faith. But here it is a question of deciding whether reason can go back to the ultimate basis of such truth and so arrive at wisdom without having recourse to faith. Augustinism denies that it can do so…"

Vernon Bourke (Bourke, p.9) summarizes the Augustinian perspective like this:

The primary understanding to which Augustine laid claim is that nothing in the world can be comprehended unless it be related to God…If you think that you understand any being without referring it to God, you are mistaken, as far as Augustine is concerned.

For Augustine, reason is not limited to science and mathematics but has a much broader scope. It can handle any concepts and definitions including those that originate in God-given intuitive knowledge and those that arise from divine revelation. Its scope includes those areas that the Enlightenment excludes such as religion, ethics, values, justice, culture, and reflections on origins. Thus, even though it is broader in some ways, it is limited – it cannot penetrate to the bases of truth without faith and faith comes by grace.

However, while basic mathematical ideas are eternally part of God's intelligence, reason has a different ontological status. God does not need to pursue a sequence of logical steps in order to come to an understanding of something; rather all truth is immediately present to him. Thus reason is part of the created order.

Augustine discusses the origin and nature of reason in *The City of God* (Bettenson, p.1072).

It is God who has given man his mind. In the infant the reason and intelligence in the mind is, in a way, dormant, apparently non-existent; but of course, it has to be aroused and developed with increasing years. And thus the mind becomes capable of knowledge and learning, ready for the perception of truth, and able to love the good. This capacity enables the mind to absorb wisdom, to acquire the virtues of patience, fortitude, temperance, and justice, to equip man for the struggle against error and all the evil propensities inherent in man's nature, so that he may overcome them because his heart is set only on that Supreme and Unchanging Good. Man may indeed fail in this; yet, even so, what a great and marvelous good is this capacity for such good, a capacity divinely implanted in a rational nature!

In a recent talk at the University of Regensburg in Germany, Joseph Ratzinger, currently Pope Benedict XVI, spoke on faith and reason, expressing a classical Christian position (Benedict).

…the faith of the Church has always insisted that between God and us, between his eternal Creator Spirit and our created reason there exists a real analogy, in which - as the Fourth Lateran Council in 1215 stated - unlikeness remains infinitely greater than likeness, yet not to the point of abolishing analogy and its language.

Benedict goes on to argue that to act inconsistently with reason is to act inconsistently with the nature of God. Augustine would almost certainly have embraced this position.

From Augustine till 1850

The central premise of the Augustinian approach, that in dealing with mathematics, we are dealing with divine ideas, was widely held for nearly 1500 years. Thomas Aquinas differed on some of the details – for example, he viewed mathematical knowledge as abstraction from experience rather than as *a priori* knowledge as did Augustine – but he accepted the central premise. The only serious challenge in the medieval period was nominalism – the philosophy that there are only individuals and particulars, and that abstract concepts in general have no real referents. Nominalism is most often associated with the work of William of Ockham in the first half of the fourteenth century and while the debate between realism and nominalism became the primary philosophical conflict of the late medieval period, nominalism never predominated.[3]

One frequently quoted example of the Augustinian perspective is this statement by Johannes Kepler,

> *I was merely thinking God's thoughts after him. Since we astronomers are priests of the highest God in regard to the book of nature, it benefits us to be thoughtful, not of the glory of our minds, but rather, above all else, of the glory of God.*[4]

However, a major shift in this way of seeing mathematics began in the early seventeenth century with Descartes; most scholars date the beginning of the Enlightenment to his work. To Descartes, the world was mechanistic and its operations could be defined in mathematical language. He was very optimistic about the capabilities of mathematics; although he professed to be a religious believer, he began a movement toward seeing mathematics not as originating in God's mind but as simply being a property of nature. He wrote (Descartes, p.21)

> *The long chains of simple and easy reasonings by means of which geometers are accustomed to reach the conclusions of their most difficult demonstrations led me to imagine that all things, to the knowledge of which man is competent, are mutually connected in the same way, and that there is nothing so far removed from us as to be beyond our reach, or so hidden that we cannot discover it, providing only that we abstain from accepting the false for the true, and always preserve in our thoughts the order necessary for the deduction of one truth from another.*

Another very influential perspective that vied with Augustine's was articulated by Immanuel Kant. He took a position intermediate between realism and nominalism, sometimes called empirical realism, which viewed abstract ideas as having a real existence but only as entities shared among human minds. This removed mathematics even further from a grounding in God.[5]

Nevertheless, in spite of the fact that alternatives were debated, the Augustinian perspective was still widely extant as recently as the mid-nineteenth century. For instance, in a recent work, a historian, Daniel Cohen writes (Cohen, p.8),

> *Edward Everett, the New England politician, Harvard administrator and orator, summarized the feelings of many early Victorian clergymen and mathematicians alike in an 1857 lecture at the inauguration of Washington University in St. Louis. He eloquently announced to the spectators, "In the pure mathematics we contemplate absolute truths, which existed in the Divine Mind before the morning stars sang together, and which will continue to exist there, when the last of their radiant host shall have fallen from heaven." Much of Everett's audience surely nodded in agreement with his lofty assessment of mathematics.*

Furthermore, many well-known mathematicians saw a close connection between mathematics and their faith. For example, after George Boole's death in 1864, his widow, Mary Boole, wrote (Cohen, p.77),

> *Mathematics had never had more than a secondary interest for him; and even logic he cared for chiefly as a means of clearing the ground of doctrines imagined to be proved, by showing that the evidence on which they were supposed to give rest had no tendency to prove them. But he had been endeavoring to give a more active and positive help than this to the cause of what he deemed pure religion.*

The post-1850 movement toward secularization

Beginning about 1850, though, a movement began that culminated in a virtual disappearance of the Augustinian perspective from the mathematical community. Several factors contributed to this process.[6] One was the proliferation of apologetics using mathematics to argue for theological truths. These were often poorly formulated and an embarrassment to the mathematical community. So the community tended to distance itself from them. For example, one such apologetic "proves" the Athanasian Creed – the creed is Trinitarian, but emphasizes God's fundamental unity and his infiniteness. The argument amounts to the statement $\infty + \infty + \infty = \infty$. Some significant events that occurred within mathematics also contributed. The principal factors were the growing rigorization of mathematics, its increasing professionalization, and the advent of non-Euclidean geometry.

Toward the end of the eighteenth century, mathematicians began to be aware of some inconsistencies in the treatment of calculus. Whereas Lagrange attempted to base all of calculus on power series, functions were discovered whose power series did not converge to the original function. Augustin-Louis Cauchy addressed these problems in part by developing the now familiar ε-δ definition of limit and the definition of continuity based on it. He went on to "prove" rigorously that the sum of an infinite series of continuous function is continuous. When counter-examples to this assertion were found, even more rigorous arguments were developed. Thus rigor was very effective in resolving these and a number of other problems in analysis. But some mathematicians objected to this growing rigorization on grounds that it made mathematics remote and inaccessible to non-specialists (Richards). Others argued that this was not at all undesirable – mathematicians had to deal with too many circle-squarers, angle trisectors, and other amateur mathematicians. But because of it effectiveness in resolving fundamental difficulties, rigor was established as the norm for carrying out mathematical arguments, thus making mathematics considerably more professionalized. Descartes had already separated mathematics from God; Kant separated it from nature; it now became more technical and was separated from popular understanding.

The advent of non-Euclidean geometries continued this process.[7] In fact, Bertrand Russell wrote (Cohen, p.180),

> *It has gradually appeared, by the increase of non-Euclidean systems, that Geometry throws no more light on the nature of space than Arithmetic throws on the population of the United States…Whether Euclid's axioms are true, is a question as to which the pure mathematician is indifferent…The [modern] geometer takes any set of axioms that seem interesting and deduces their consequence.*

Russell's intellectual predecessor in studying mathematical logic, Gottlob Frege, saw logic as expressing the laws of human thought. Russell agreed. (Cohen, p.179)

> *He marveled at giants of recent logic and mathematic such as Weierstrass, Dedekind, and Cantor…Yet Russell's praise of these pure mathematicians only superficially mimicked early Victorian odes. What made these theorists so inspiring, he argued was not that they unveiled God's thoughts, but that they gave us a much better understanding of the human mind and the concepts we use without reflection.*

From Russell's perspective, his crowning achievement in mathematics was his demonstration in his *Principles of Mathematics* (1903) that the natural numbers can be derived from logic. The natural numbers were the principal example that Augustine had used to locate mathematics in the mind of God. Thus Russell had completed a process that Descartes had begun – of establishing mathematics as an autonomous entity with an identity that depended only on human beings. Perhaps the most influential mathematician of the twentieth century, David Hilbert, succinctly expressed this perspective: (Hilbert)

> *The fundamental idea of my proof theory is none than to describe the activity of our understanding, to make a protocol of the rules by which our thinking actually proceeds…Already at this time I would like to assert what the outcome will be: mathematics is a presuppositionless science. To found it, I do not need God or the assumption of a special faculty of our understanding…or the primal intuition of Brouwer…or finally, as do Russell and Whitehead, axioms of infinity, reducibility, or completeness.*

Thus by the early years of the twentieth century, the perspective that saw mathematics as the study of divine ideas had largely disappeared.

Problems and questions

Realism, such as Augustine articulates, has encountered some serious obstacles since the late nineteenth century. In fact, much of the twentieth century philosophy of mathematics represents a mixture of attempts by some scholars to replace realism and others to recover it.

For example, Russell's paradox presents one such problem. If God is infinite and omniscient and if abstractions such as sets have a real existence in the mind of God, then presumably the set of all sets also exists in God's mind. God can also easily make the distinction between those sets that are members of themselves and those that are not. Thus Russell's paradox – that the set of all sets that are not members of themselves both is and is not a member of itself – would say that God's thoughts are contradictory.

Christopher Menzel has developed a plausible response to this paradox and others related to it (Howell, Chapter 3). Essentially, Menzel's response is that Russell's paradox is based on an impredicative definition. That is, the process of forming a set is a collecting action – the existence of the set {1, 2, 3} depends on the prior existence of 1, 2, and 3. Thus if S denotes a collection of sets, the sets in the collection have to have been well-formed prior to being collected. Asking whether S is a member of itself assumes that S is well-formed even though its elements have not been collected. So the "paradox" is the result of an earlier inconsistent assumption. Menzel then models set formation as just such a collecting activity in the mind of God. Nevertheless, a great deal of work needs to be done to test how well the Augustinian model can stand up to the critiques of realism in the recent philosophy of mathematics.

There are spiritual and intellectual problems with the secular approach as well. In calling attention to these, I'm not attempting to turn the clock back to the pre-Enlightenment era. Nineteenth and twentieth century mathematics represent monumental achievements that Christians can strongly affirm. Rather it's that Christians "read" the significance of those achievements differently.

Consider first the process of secularization that began with Descartes and culminated in Russell's assertion that mathematics and logic represent nothing beyond the laws of human thought. There is a spiritual problem here. The declaration of human autonomy from God is the essence of sin; thus a plausible Christian response to Russell and Hilbert's assertions of the autonomy of mathematics is to see them as additional examples of this familiar phenomenon. There is also an intellectual problem – if one accepts the notion that mathematical ideas existed in the mind of God prior to creation, the fact that mathematics can be separated from the physical universe is no surprise. Also if one believes that the capacity to use reason was built into human minds by God, the fact that logic expresses certain patterns or laws of human thought is equally unsurprising. These observations simply point back to the creator. The implicit premise in Russell and Hilbert's thought:

If mathematics can be cut loose from moorings in anything external to itself, then it doesn't need God or anything external to human thought.

is easily refutable if one starts from Augustinian premises.

Further weaknesses of the secular perspective are that it is unable to account for some important facts: that humans have an extensive capability to engage in mathematics, that mathematics provides such certitude, and that mathematics is so effective in describing physical reality. The Enlightenment perspective also fractures human thought. That is, the Enlightenment defines reason as empiricism and mathematics. But this view excludes ethics, culture, justice, values, and religion, all of which are central to the life of the human community and, from a Christian perspective, fall well within the scope of reason. Thus rather than glorifying human thought, secularism drastically reduces the scope of reason. In mathematics, the consequences of this have been severe: applied mathematics has been largely separated from ethics;[8] the history and philosophy of mathematics are rarely taught in mathematics graduate programs; any attempt to establish a connection between mathematics and divine thought has come to be widely seen as virtually irrational.

Conclusions

The Augustinian view of mathematics has much to commend it. It's inspiring – from this perspective the capacity to do mathematics is a gift of God, its content originates in God, a mathematical career is a calling to discover God's wonders, and it leads both to service of his kingdom and to worship.

It also leads Christians to work for a more holistic view of mathematics. Twenty-first century mathematics has already moved somewhat away from the decontextualized logicism of Russell and the formalism of Hilbert. Nevertheless, their legacy still maintains a significant hold on the discipline. A Christian perspective points toward a mathematics that is far more inclusive not merely of applied mathematics but of history, philosophy, ethics, and culture and that sees mathematics as one valuable thread in a much broader fabric of divine and human thought.

Endnotes

[1] Two translations are currently widely available. See Benjamin and Williams. Quotations here are from Williams' translation.

[2] The Scripture referred to here is Ecclesiastes 7:25. Augustine quotes this verse, presumably from the Septuagint. Williams translates his quotation as "I went around, I and my heart, that I might know and consider and seek after wisdom and number." The King James translation reads, "I applied mine heart to know, and to search, and to seek out wisdom, and the reason of things…" The New International Version translates this verse as, "So I turned my mind to understand, to investigate, and to search out wisdom and the scheme of things."

[3] See, for example, the Catholic Encyclopedia's article on the History of Dogmatic Theology at http://www.newadvent.org/cathen/14588a.htm and some of the links that can be reached from it.

[4] This statement is often attributed to Kepler, but never with the source. I have included it here because it is so widely quoted and such a clear example of the Augustinian perspective.

[5] A thorough, well-written discussion of these issues can be found at http://www.friesian.com/universl.htm.

[6] Cohen provides an excellent and detailed study of the process and the forces that drove it. My brief remarks here are a summary of Cohen.

[7] Kant was an "anti-realist." He believed Euclidean geometry was synthetic knowledge – a form imposed on the world by our minds. Some historians of mathematics have viewed Kant's idea that Euclidean geometry is an integral part of human thought as an impediment to the development of non-Euclidean geometry. But, in another sense, it helped lay the groundwork for it – that is, it advanced the project of separating mathematics from nature. From a Kantian perspective, Euclidean geometry no longer had a tie to nature that gave it a special claim on truth.

[8] For example, operations research, the study of mathematical models of decisions, would appear to lend itself well to the inclusion of ethics. But in fact, ethical issues are not addressed in OR textbooks and are rarely mentioned in the literature of the field. In the early days of OR, the role of ethics was extensively debated. George Dantzig and others who wanted ethics excluded ultimately prevailed on the grounds that they wanted the new discipline of OR to be a "scientific" discipline. Most other branches of applied mathematics also rarely mention ethics. Statistics, however, includes extensive work on ethics largely due to the influence of social scientists.

References

Benedict XVI, aka Joseph Ratzinger, *Faith, reason and the university: memories and reflections*, www.guardian.co.uk/pope/story/0,,1873277,00.html

Benjamin, Anna S. and Hackstaff, L.H., translators, Saint Augustine, *On Free Choice of the Will*, Prentice-Hall, 1964

Bettenson, Henry, translator, Saint Augustine, *The City of God*, Penguin Classics, 2003

Bourke, Vernon J., *Augustine's View of Reality*, Villanova Press, 1964

Cohen, Daniel J., *Equations from God: Pure Mathematics and Victorian Faith*, The John Hopkins University Press, 2007

Davis, Philip J. and Hersh, Reuben, *The Mathematical Experience*, Houghton Mifflin Company, 1981

Descartes, Rene, *The Method of Rightly Conducting the Reason and Seeking Truth in the Sciences*, 1637, translated by John Veitch, LLD

Ernest, Paul, *Social Constructivism as a Philosophy of Mathematics: Radical Constructivism Rehabilitated?*, http://www.people.ex.ac.uk/PErnest/soccon.htm

Gilson, Etienne, *The Christian Philosophy of Saint Augustine*, Victor Gollanz, Ltd, 1961

Hilbert, David, *The Foundations of Mathematics*, 1927, http://www.marxists.org/reference/subject/philosophy/works/ge/hilbert.htm

Howell, Russell W. and Bradley, W. James, *Mathematics in a Postmodern Age: A Christian Perspective*, Wm. B. Eerdmans Publishing Company, 2001

Kline, Morris, *Mathematics in Western Culture*, Oxford University Press, 1953

Richard, Joan, *The Rigorous and the Natural in Eighteenth Century Mathematics,* Proceedings of the Biennial Conference of the Association of Christians in the Mathematical Sciences held at Wheaton College May 29 - June 1, 1991

Williams, Thomas, translator, Saint Augustine, *On Free Choice of the Will*, Hackett Publishing Company, Inc., 1993