Ideas at the Intersection of Mathematics, Philosophy, and Theology by Carlos R. Bovell
An essay review and a summary by Gene B. Chase

Carlos R. Bovell in his eleven essays, *Ideas*, sees himself as answering this one big question: "How can the history of mathematics shed light on philosophy and theology?" (81 *et alia*) In this project, he addresses this theme, but also the theme of how philosophy and theology shed light on mathematics, quite beyond the examples that I provided in (Chase, 1997).

Bovell says in his preface that *Ideas* is a collection of essays about Christian faith and mathematics. He cites a refreshing collection of authors who are not usually referenced in this JOURNAL. Despite his Reformed training at Westminster Theological Seminary and at Institute of Christian Studies, he draws freely on Catholic theologians. For example, he cites Bernard Lonergan's use of mathematics. See (Manning, 1983) for more details than Bovell gives. For example, he cites Jean Luc Marion on infinity. As I was reading these essays, my list of things that I "must" read grew as Bovell whetted my appetite. You will find the same if you want to answer his big question.

Bovell approaches his big question primarily through phenomenology. The very title of this collection of essays is borrowed from "the father of phenomenology" Edmund Husserl's *Ideas*. (Husserl, 1913) What is phenomenology? Without putting too fine a point on it, realism places reality "out there" and idealism places reality "in the mind." As examples of "too fine a point," I won't ask whether it's "*my* mind" (solipsism) or "human minds" collectively; I won't ask whether "out there" is material or Platonic. Phenomenology places reality between those two loci: in perception, which of course needs both a perceiver and a perceived. Without "too fine a point," perceptions include intuitions—insight as well as sight.

Some well-known mathematicians are phenomenologists, such as Gian-Carlo Rota, whose research spans a spectrum from linear operators on Hilbert spaces to Möbius inversion as a technique for solving combinatorial problems. See my review of Rota's book *Indiscrete Thoughts*, where he makes his phenomenology explicit. (Chase, 1998) Conversely, some well-known phenomenologists are competent mathematicians, such as Husserl himself. See Dallas Willard's definitive translation and commentary on some of Husserl's mathematics. (Husserl, 2003) Yes, *that* Dallas Willard, author of *The Divine Conspiracy*. Some scholars whom Bovell cites are hard to classify. For example, he cites Charles Sanders Peirce multiple times—mathematical logician anticipating Cantor, Peano, and Zermelo; and a phenomenologist, although often he is usually called "the father of pragmatism."

Yet in a typical course in philosophy of mathematics, phenomenology is omitted. The usual candidates to study are realism, intuitionism, formalism, and logicism, and if your text is the standard one by Stewart Shapiro, then you have to include structuralism, because Shapiro is a structuralist. (Shapiro, 2000) The present essays provide some remedy for this omission.

Bovell professes philosophy but has a strong background in mathematics. He even uses the generalized Stokes theorem as one of his examples! (If you're searching his essays electronically for the example, search for "Spivak".) Bovell probes further about things that you have probably thought about without probing them as much as he has. For example, in what sense is Aquinas' *Summa* a "spiritual Euclid" as Morris Kline calls it? Or, in what sense does 1+1+1=1 model the Trinity as a variety of papers by Christian mathematicians point out?

In my summaries of the individual essays below you will easily be able to tell which ones you will find interesting. Regular readers of this JOURNAL will find several interesting and several not interesting at all. The essays are uneven because they were prepared for varied audiences. I pitch my summaries to mathematicians, which makes the mathematics shorter, but some of the essays are heady philosophy regardless. I warn you.

I offer a chapter-by-chapter summary with modest commentary.

1 The definition of mathematics seems to limit its potential for the humanities.

This essay asks more questions than it answers. What is mathematics? How can undergraduates in the humanities be helped to appreciate mathematics? Can "real human problems" (9) be addressed with no more than pre-Calculus mathematics?

My comments: Mathematicians become mathematicians because mathematics is incredibly beautiful. Musicians labor over Bach's Toccata and Fugue in D Minor because it is incredibly beautiful. What then is the barrier for students in the humanities to enjoying mathematics in an appreciation course? Can I appreciate Bach's organ work without the years of labor in mastering the organ? Can I appreciate mathematics without the years of labor in mastering technique? Bovell suggests that it is possible but difficult.

Bovell defines mathematics as "what mathematicians do." Increasingly there are books which provide a humanistic approach to understanding what mathematicians do. See Auburn, Nasar, MacCormick, Richeson, and Singh in my references for just a few recent examples. I don't have to be able to treat elliptic curves as abelian groups in order to appreciate how encryption secures the internet or how Andrew Wiles proved Fermat's last theorem. But of course securing the internet is not the humanities, nor is Fermat's last theorem. Thus Bovell doesn't think that the answer to providing a humanistic mathematics lies in "applied" mathematics," even though, as psychologist Kurt Lewin said, "There is nothing so practical as a good theory." (1951: 169)

Here is Bovell's solution: Mathematicians should be trained to "look for linkages" as they teach mathematics. "Linkages" are much broader than applications. Bovell is his own best example. He finds linkages everywhere.

2 Ninety-nine percent of mathematics has little to no use for philosophy.

If mathematics is about doing, and philosophy is about reflecting on doing, then most people who are polled say that philosophy is relevant, but academicians don't seem to agree. Even mathematicians, whose doing is a kind of reflection, say philosophy is irrelevant. Bovell, who is a philosopher, agrees that philosophy is irrelevant! I commend him for not taking himself too seriously.

3 Some ideas on Husserl's remarks that mathematicians are not pure theoreticians

Husserl claims that mathematicians are not theoreticians, but technicians, and that theory falls to the philosopher. Bovell claims that philosophers are technicians too!

Husserl appears, according to Bovell, to want to assimilate mathematics into phenomenology. Yet Bovell claims that mathematics goes beyond intuition. (24) Husserl as a phenomenologist holds that the real world is "given through perception." (26) Nonetheless, phenomenology is a "programmatic search" (33) just as mathematics is, claims Bovell. He later cites Lakoff and Núñez favorably in support of this claim. (Chapter 9)

4 Some ideas on Heidegger and the influence of mathematics and science on metaphysics

Metaphysics studies the nature of reality in general. It's not about being, but about "being there." (German: *Dasein*) Husserl's student, Martin Heidegger, claims that we can't know "being there" unless we also understand its opposite: the nothing. Heidegger is pushing back against the emphasis on logic of the Vienna school of logical positivists and of Bertrand Russell. But logic can't discuss the nothing, because an attempt to discuss the nothing turns it into a subject, namely the "*concept* of nothing." (38) Philosophy is "impoverished" if that is attempted. (39)

"[T]he task of science is to explain human experience" (42) without inserting ourselves into the experience. The task of philosophy—to Heidegger as other phenomenologists—is to insert ourselves into the experience, the "being there," our existence. For example, Newton's laws became true "through Newton." (43) Since logic is inadequate for philosophy, then so is mathematics.

Husserl criticized Heidegger for reducing phenomenology to anthropology. Pragmatists place mathematical reality not in the external world, and not in the mind, but in the culture, hence in the "mind of the species." (45) So maybe pragmatists like C. S. Peirce do a better job of explaining Husserl than Husserl does, claims Bovell.

The roots of phenomenology lie deeper than Husserl. In Bovell's Chapter 6, Newton is quoted as a phenomenologist: "I frame no hypotheses; *for whatever is not deduced from the phenomena* [sic], is to be called an hypothesis." (57)

5 A comparison between Euclid and Aquinas and a question of method

Morris Kline calls Aquinas' *Summa* the "spiritual Euclid" (Kline, 96). Bovell however claims that "Euclid and Aquinas are rarely considered together." (47) This may be because Aquinas reasons analogically, "cast[] in syllogistic molds." (54) That doesn't disqualify mathematics, for "[t]here is a subjective element in mathematics" too. (47)

In what sense then is the *Summa* a "spiritual Euclid"? Euclid reasons deductively (synthetically). But of course proofs are discovered analytically. So Bovell proposes to read Aquinas backward, just as we might read Euclid backward from the Law of Pythagoras to find out how its proof may have been discovered. Euclid's presentation burns his bridges behind him so to speak (including the *pons asinorum* I might add!).

Bovell concludes that Aquinas's argument is more like Euclid's Quod Erat Faciendum than his Quod Erat Demonstrandum—more like a construction than a deduction. This is an excellent discussion!

One place where Aquinas broke from Aristotle was in terms of God's perfection. For Aristotle, God's perfection was beyond the visible creation in the outermost heavens; for Aquinas, it was in the visible creation.

(Bovell bases his discussion on *Summa contra Gentiles*, whereas Morris Kline is referring to *Summa Theologica*, but the latter is a revised and expanded version of the former, so which is cited doesn't matter for this discussion.)

Bovell could well incorporate the powerful insights of Imre Lakatos (1976) on how definitions get modified to make theorems true. This essay complements Lakatos perfectly.

6 On the "good and necessary consequence" clause in the Westminster Confession of Faith (1647)

Do not be surprised that the Westminster Confession sets scripture down as axiomatic, given the cultural milieu of its time to conform every discipline to a mathematical mold. Yet if scripture is axiomatic, then it can't generate "the whole counsel of God" which it claims to do. This is so whether we make an analogy with the Gödelian paradox of mathematics or with the Kolmogorov-Chaitin complexity of computer science. An example of a theorem which cannot be deduced from scripture is the Westminster Confession of Faith itself! (62–63)

Theology needs insight and conversion as Bernard Lonergan argues; theology needs analogy and dialectic as Aquinas argues. Mathematics as a whole can be an analogy for philosophical theology, but mathematics cannot carry the weight of all of philosophy. Leibniz had already criticized Descartes along these same lines. (65)

7 Thoughts on supernaturalism and its irrelevance for science and mathematics

This is a defense of methodological naturalism and its irrelevance for science.

My comments: As for the "and mathematics" of the title, that is not even addressed in this essay, despite its being repeatedly stated. It may be true that mathematics is fallible, says Bovell. Certainly non-Euclidean geometries and everywhere non-differentiable functions are examples of mathematics that were disbelieved before the 19th C. The future of mathematics is open. Matsumoto said as much in this JOURNAL in 2006 (Matsumoto).

8 Thoughts on the Intermediate Value Theorem and the "knowledge-boundary" problem

The so-called Gettier problem challenges whether knowledge of a proposition *p* is equivalent to justified true belief about *p*. Some fallibilists argue that because the boundary between knowing *p* and not knowing *p* cannot be found, it is not even meaningful to say that *p* can be known. But if by "found" we mean "known" then that's an infinite regress.

The intermediate value theorem (IVT) talks about a boundary that can't be known but can be proven to exist. Does that analogy help us any? The IVT is an "existence" theorem, not a knowledge theorem. Furthermore (I claim, not Bovell) it's an existence of a rather vapid sort: a proposition with a leading quantifier $\exists$, not a metaphysical existence. And (Bovell claims) the analogy depends on epistemic justification being a continuous quantity.

This essay is really just some playful musings delivered at a mathematics meeting.

9 On the associative property of addition and its application to the Godhead

The doctrine of the Trinity is intelligible but not comprehensible. Christian mathematicians have sometimes used the so-called equation 1 + 1 + 1 = 1 to illuminate the doctrine without looking hard at what that would mean. Philosophers as varied as Boethius (fl. 500 CE) and Bertrand Russell have wrestled with identity of numbers, in the case of Boethius, precisely in the context of discussing the Trinity. As Aristotle already said (roughly), you can't add apples and oranges unless you reclassify them as both fruit. Then fruit + fruit = 2 fruit.

The various Christian doctrines of Divine persons; of perichoresis, that is, of interpenetration of the Divine persons (118); of Divine simplicity; and of Divine substance complicate the discussion of the equation, not to mention what we mean by "=" — whether identity or some other equivalence relation. Likewise "+" can be more generally logico-algebraic or more specifically numerical. If we replaced the equation with $x + x + x = x$, we'd have a clear solution of $x = 0$, which is not helpful to our discussion.

Now in math a + b + c is a shorthand for (a + b) + c, so an associative law inevitably comes into play, since "+" is a binary operation. That was how it was historically with the church: Jesus was affirmed to be God before the Holy Spirit was recognized as a separate person. It's not that we couldn't have a ternary relationship +(a,b,c), but there seems to be something innate in humans to understand pairing more easily. Another problem is that we think in time, but God stands outside of time. (96 n. 31)

Bovell concludes: We may be stuck with 1 + 1 + 1 = 1 being either vague or erroneous.

10 Remarks on the search for an infinite God in the philosophy of Edmund Husserl

None of Georg Cantor's infinities are the Absolute Infinite, as Cantor himself maintained, but are inspired by God as the only [absolute] infinite. (102) Cantor credits St. Augustine's *City of God*, but here Bovell shows that Descartes says the same thing about God. (102)

Husserl draws on Cantor "framing the phenomenological question of the infinite horizon of absolute consciousness . . . via developments in mathematics." (105) Husserl admits that the "catalyst" of his phenomenology was "a religion . . . able to bear the existential brunt of the mathematical problems that occupied him," but Husserl explicitly says that he's not a Christian philosopher. (124) Even so, Husserl says that a phenomenological approach will deepen your appreciation for "God and God's word, man in search of God, living as a child of God" to a degree "comparable . . . to a religious conversion." (108)

Contemporary Catholic phenomenologist and Cartesian scholar Jean Luc Marion also argues that the "infinite horizon [of conscious understanding] describes an encounter with God." When Descartes says that God is infinite, he is not stating it as a negative, but as a positive. (99). Bovell ties Marion and Husserl together: "Husserl's discovery of the absolute ground of the world is precisely what Marion indicates: a transcendental, phenomenological glimpse of the infinite Creator of the world." (100)

Marion says that "the freedom found in the *ego* follows from the infinity of its will, which itself depends on its likeness to God." "This," claims Bovell, " is precisely the theme that can be discerned in Husserl's writing." (103) Husserl sees Cantor's work as a "historical moment[] of spiritual transformation" of the culture of ideas (105).

Bovell concludes: "[P]erhaps the phenomenological reductions are also meant to lead one to . . . a groping for the infinite God." (109)

11 Observations regarding the Kalam argument and its disavowal of actual infinites

Since Aristotle's time actual and potential infinity have been distinguished. Bovell here argues for a finer distinction. Is the actual infinity in mathematics or in space-time? For example, theologian William Lane Craig argues that our minds can conceive of an actual infinity existing, perhaps in mathematics, but not in physical reality. (112 n. 6) Bovell cautions us to make sure that we don't change the meaning of existence from one part of the argument to another as we reason about actual infinity.

Take for example the Kalam argument for the existence of God, which to quote Craig is:

> "I. Everything that begins to exist has a cause of its existence.
> II. The universe began to exist.
> III. Therefore the universe has a cause of its existence." (110 n. 1)

Bovell claims that Craig's notion of existence may change from I to II to III. And these infinities that Craig discusses surely do not exhaust all the particular infinities that there might be. (117–118)

For example, Augustine said that there is no infinite past time (see Craig's II above) because God created time, and there is no infinite space because there is no space beyond the cosmos. (116) But according to Augustine, there is an infinite because God is infinite. Furthermore, contrary to Augustine, other Christian theologians suggest that the universe might be co-eternal with God, indeed suggest that this does not diminish God's creation of the universe.

There is a distinction between the universal notion of the infinite and specific instances of the infinite. Bovell tentatively offers an anti-nominalist stance on the existence of the infinite: "Perhaps universals [like an infinity] are present in the physical world, precisely in the case of every instantiation without its existing [in some Platonic realm] apart from them?" (119)

Bovell offers two examples of the infinite being found in the finite. First, mathematically, finite irrational numbers like □ are infinite repeating decimals. Second, Christian orthodoxy maintains that the infinite Son of God became the finite Jesus. The latter is paradoxical, but we shouldn't rule out actual infinities because of paradoxes. Paradoxes about actual infinity do not mean that actual infinity doesn't exist. They mean merely that human apprehension of actual infinities is limited.

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