# Does Mathematical Beauty Pose Problems for Naturalism?[1]

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Numerous events occurred in 1960 whose effects could hardly have been predicted at the time: several African Americans staged a "sit-in" at a Greensboro lunch counter, the Soviet Union shot down Gary Powers while he was flying a U2 spy plane, the US FDA approved the use of the first oral contraceptive, AT&T filed with the Federal Communications Commission for permission to launch an experimental communications satellite, and four Presidential debates between John Kennedy and Richard Nixon aired on national television.

Less well known was the publication of a paper by the physicist Eugene Wigner. Appearing in *Communications in Pure and Applied Mathematics*, a journal certainly not widely read by the general public, it bore the mysterious title "The Unreasonable Effectiveness of Mathematics in the Natural Sciences." (Wigner, 1960) Like our cultural examples of the 1960's, it has had effects beyond what most people would have imagined. Our purpose here is to tease out some strains of an important question that has emerged from Wigner's work.

Wigner begins with a story about two friends who were discussing their jobs. One of them, a statistician, was working on population trends. He mentioned a paper he had produced, which contained the Gaussian distribution equation near the beginning. The statistician attempted to explain the meaning it, as well as other mathematical symbols. His friend was a bit perplexed, and was not quite sure whether the statistician was pulling his leg. "How can you know that?" he repeatedly asked. "And what is this symbol here?"

"Oh," said the statistician, "this is pi."

"What is that?"

"The ratio of the circumference of a circle to its diameter."

"Well, now you are pushing your joke too far. Surely the population has nothing to do with the circumference of the circle."

Wigner uses this story to introduce two issues: (1) the surprising phenomenon that we have used mathematics so often to build successful theories; (2) the nagging Kuhnian-like question,

> *How do we know that, if we made a theory which focuses its attention on phenomena we disregard and disregards some of the phenomena now commanding*

*our attention, that we could not build another theory which has little in common with the present one but which, nevertheless, explains just as many phenomena as the present theory?* (Wigner, 1960)

Regarding Wigner's first point, he concedes that much of mathematics, such as Euclidean Geometry, was developed because its axioms were created on the basis of what appeared to be true of reality. From this viewpoint the applicability of mathematics to the physical world is hardly surprising. But how much of mathematics actually progresses in this manner? A strong argument can be made that other notions guide the formation of a large body of higher mathematical theories.

Take the field of complex analysis as just one example. In the 1500's the notion of $\sqrt{-1}$ seemed odd to mathematicians. At that time, negative numbers by themselves were still being treated with some suspicion, so taking square roots of them was all the more problematic. But mathematicians kept using their imagination and pressed forward. Serious investigation of complex numbers dates to the mid-fourteenth century, when Scipione del Ferro of Bologna solved the depressed cubic equation, which is a cubic equation without an $x^2$ term, such as $x^3 - 15x - 4 = 0$. The solution was independently discovered some thirty years later by Niccolo Fontana. Girolamo Cardano subsequently extended the "Ferro-Fontana" formula to obtain a solution of the general cubic equation, which he published in *Ars Magna* in 1545. Then, in 1572, Rafael Bombelli used Cardano's published work to interpret the form of solutions to some depressed cubic equations. Prior to Bombelli, these forms had been impossible to decipher.

Bombelli derived his solutions by using complex numbers. For example, Bombelli's techniques, when applied to the depressed cubic equation $x^3 - 15x - 4 = 0$, yielded a solution of the form $x = (2 + i) + (2 - i)$. Simple arithmetic then gave $x = (2 + i) + (2 - i) = 4$. That $x = 4$ was a correct solution to the original equation was indisputable, as it could be checked easily. However, it was only arrived at via a detour through the uncharted territory of complex numbers.

The story that details the entire development of complex numbers is quite intricate, and it wasn't until the end of the nineteenth century that they became firmly entrenched in the corpus of mathematical literature. For our purposes, it is important to note that complex numbers were studied because they were useful for mathematical and not for physical purposes. To be sure, solving equations had the potential to be of great practical value, but no physical phenomenon guided the investigation of complex numbers. Success came mainly from abstraction, and the manipulation of mathematical symbols in accordance with specified rules of algebra.

Complex numbers, however, now play a pivotal role in helping physicists understand the quantum world. For Wigner, it is not just the utility of complex numbers in quantum mechanics that is surprising; it is that, time and time again, leaps of theory seem to be successfully guided by mathematical formalisms rather than experimentation. According to Wigner,

> *Quantum mechanics originated when Max Born noticed that some rules of computation, given by Heisenberg, were formally identical with the rules of computation with matrices. ... Born, Jordan, and Heisenberg then proposed to replace by matrices the position and momentum variables of the equations of classical mechanics. ... The results were quite satisfactory. However, there was ... no rational evidence that their matrix mechanics would prove correct under more realistic conditions. As a matter of fact, the first application of their mechanics to a realistic problem, that of the hydrogen atom, was given several months later, by Pauli. This application gave results in agreement with experience. This was ... understandable because Heisenberg's rules of calculation were abstracted from problems which included the old theory of the hydrogen atom. The miracle occurred only when matrix mechanics ... was applied to problems for which Heisenberg's calculating rules were meaningless. Heisenberg's rules presupposed that the classical equations of motion had solutions with certain periodicity properties; and the equations of motion of the two electrons of the helium atom, or of the even greater number of electrons of heavier atoms, simply do not have these properties, so that Heisenberg's rules cannot be applied to these cases. Nevertheless, the calculation of the lowest energy level of helium ... [agreed] with the experimental data within the accuracy of the observations, which is one part in ten million.* (Wigner, 1960)

"Surely," Wigner concludes, "in this case we 'got something out' of the equations that we did not put in."

Wigner cites other examples: Newton's law of motion, formulated in terms that appear simple to mathematicians, but which proved to be accurate beyond all reasonable expectations; quantum electrodynamics; and the pure mathematical theory of the Lamb shift. He finally concludes his paper with the following observation.

> *The enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious, and ... there is no rational explanation for it. ... The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning.* (Wigner, 1960)

In 1980 Richard Wesley Hamming took up the issues raised by Wigner, and offered four "partial explanations" that could account for the applicability of mathematics. (Hamming, 1980) Following is a brief survey of Hamming's ideas.

First, we see what we look for. Mathematicians craft postulates so that they will produce theories that conform to their prior observations. The Pythagorean theorem, Hamming claims, drove the formation of the geometric postulates, and not vice versa. Furthermore, if we insist on looking at the world through a mathematical lens, it is not entirely surprising that we wind up describing the world in mathematical terms.

Second, we select the kind of mathematics to use. Hamming simply means here that we adopt the mathematical theories that seem, ahead of time, to be a good fit for a physical phenomenon being investigated. The same type of mathematical theory does not work everywhere; different theories are selected in accordance with the phenomenon they seem to describe. Because we habitually force mathematics onto particular situations, it is only natural that we subsequently find mathematics in general to be so widely applicable.

These first two explanations are quite similar, although part of the first one is more in line with Wigner's Kuhnian-like question, which we will discuss later on. Regarding the selection of mathematics to fit our perception of reality, we have already acknowledged that some of mathematics develops this way. To the extent that not all applicable mathematical theories are generated out of concern for applicability, further work must be done, and Hamming indeed gives some additional food for thought.

Hamming's third response is that science in fact answers comparatively few problems. To the extent that this assertion is true, the less of a miracle the success of mathematics would appear to be. Wigner, as a physicist, certainly lived with mathematics as an indispensable tool, but other sciences do not share this same reliance on mathematics. Biology, it is often said, has not yet been successfully dissected by the mathematical scalpel.

This position may have warrant, but upon reflection, some doubt can be cast as to whether it is entirely correct. Indeed, a great deal of mathematical attention has been focused of late on biological questions. A colleague of mine, for example, has looked at Coxeter Groups as models for DNA similarity. Knot theory has also had some success in the classification of DNA strands according to how they crinkle up under certain conditions. At the January 2006 annual joint meetings sponsored by the American Mathematical Society and the Mathematical Association of America, the prestigious Josiah Willard Gibbs Lecture was given by mathematical biologist Michael A. Savageau (University of California, Davis). His lecture, entitled *Function, Design, and Evolution of Gene Circuity*, was laced with mathematical models. It should be noted that this lecture is the only one on the four-day program that has no competing presentations. Thus, the importance of this area to the mathematical community is unquestioned, and therefore it seems quite likely that mathematics will play a significant role in at least some areas of future biological studies. Indeed, there now exist various societies worldwide whose task is to investigate mathematical applications in biology.

Certainly, though, Hamming has a point in that mathematics does not permeate all of science, at least to the extent that it does in physics. Even if we grant the argument that Hamming proposes here, the success of mathematics in physics itself is something that cannot be dismissed simply by pointing to slower progress in other areas. Are there any good explanations that fully account for this success?

One obvious candidate is Hamming's final suggestion, that the evolution of man provided the "model," meaning the explanation for why humans are able to mathematize the physical universe. At face value this claim may appear to be plausible, but it is not fleshed out beyond Hamming's comment that, "Darwinian evolution would naturally select for survival those competing forms of life which had the best models of reality in their minds—'best' meaning best for surviving and propagating." (Hamming, 1980, p.89) It is interesting to note that Hamming concludes with the following remark.

> *If you recall that modern science is only about 400 years old, and that there have been from 3 to 5 generations per century, then there have been at most 20 generations since Newton and Galileo. If you pick 4,000 years for the age of science, generally, then you get an upper bound of 200 generations. Considering the effects of evolution we are looking for via selection of small chance variations, it does not seem to me that evolution can explain more than a small part of the unreasonable effectiveness of mathematics.* (Hamming, 1980, p. 89)

This observation hardly seems compelling. Just as an inclined block needs a critical slope to overcome its friction and start sliding, and once the sliding begins it proceeds rather rapidly, so too one might argue that once science started it progressed quickly, but any evolutionary development that occurred before this explosion was critical, and cannot be discounted.

But evolutionary accounts have problems as well. In fact, with respect to explaining our ability to apply higher mathematics to the physical world, they are sparse at best. In what follows we briefly examine three approaches that have been put forth as viable evolutionary accounts for various forms of cognition. Their proponents do not necessarily intend that their arguments should apply to mathematical cognition, so criticisms levied against them must be taken with the requisite grain of salt.

The first explanation can be called the sexual selection hypothesis, as argued by Geoffrey Miller. He claims that excessive capacities or acquisition of resources of any kind may be a sexual display. If you have the energy or time or intrinsic capacity to do things that do not have direct adaptive value—carrying around a set of antlers that are so big they are more of a detriment than a defense, or a peacock walking around with a big colored tail, or possessing artistic or mathematical brains that are more than we need to solve the problems of survival—then that energy or time or intrinsic capacity by itself may attract mates. (Miller, 2001)

Physical attributes certainly seem to have some role in mate attraction, and artistic brains may as well insofar as they enable people to make attractive artifacts for display. The argument for mathematical brains, however, does not seem to hold up as well. Although Miller does not specifically address the question of why mathematical reasoning is successful, there has been speculation that his thinking might be relevant. Miller states,

> *The healthy brain theory suggests that our brains are different from those of other apes not because extravagantly large brains helped us to survive or to raise off-*

*spring, but because such brains are simply better advertisements of how good our genes are. The more complicated the brain, the easier it is to mess up.* (Miller, 2001, p. 104)

But how would a larger brain be evident, and how would one somehow conclude that this excess is evidence of good genes? In this regard one is reminded of the Gary Larson cartoon where, on a desert island, two men compete for the attention of women. The one winning the day is the one who is able to produce, on chalk board, a more impressive array of mathematical equations. These speculations, then, while certainly not disprovable, seem to have no good evidence in their support, at least insofar as they might relate to mathematical thinking.

Next is what we might call the module approach, as argued by Stephen Mithen. Mithen writes from the perspective of an anthropologist, and has an enormous amount of archaeological data upon which to draw. His thinking is that integrative and higher level (meta) cognitive processes grew out of the unification of specific evolutionary modules, such as a module for tool use, or a module for interpersonal relations. These modules seem to coincide with spurts of brain enlargement, which are caused by a variety of factors. For example, “In general larger animals have larger brains, simply because they have more muscles to move and coordinate.”(Mithen, 1996, p. 200) Mithen further argues that in humans (and only in humans) we also find a structure on top of modules—a general purpose rationality. Says Mithen, “In summary science, like art and religion, is a product of cognitive fluidity. It relies on psychological processes which had originally evolved in specialized cognitive domains and only emerged when these processes worked together.” (Mithen, 1996, p. 215)

This last approach has been debated extensively. For example, Alvin Plantinga’s celebrated essay, “An Evolutionary Argument against Naturalism” claims that rationality is very unlikely a quality produced by survivability. (Plantinga, 1993) Plantinga’s approach, as he acknowledges, is similar to that found in C. S. Lewis’s Miracles. (Lewis’s argument, incidentally, was recently enhanced by Victor Reppert in his book *C. S. Lewis’s Dangerous Idea.*) The thrust of their thinking is that you cannot get (or are very unlikely to get) rationality out of a causally closed system that works solely on the basis of blind chance physical interactions operating in accordance with a survival of the fittest paradigm.

This brings us to the byproduct hypothesis, as exemplified by Pascal Boyer, who indirectly argues against Lewis’s and Reppert’s view, at least as applied to religious cognition. His main thesis is that many higher cognitive religious functions may not be evolutionary adaptations at all. Instead, they may be byproducts of things that are adaptive, and just piggyback on the adaptiveness of these other capacities. For Boyer, such cognition comes from many sources, which explains why religious claims, taken as a whole, produce so many false conclusions. (Boyer, 2001)

Although Boyer is only speaking of religious sensibilities, in stating this view he seems to wave a magic wand and categorically pronounce that this piggyback model *may* be the case. On the one hand, if the piggyback model holds up, one may well be justified in extending Boyer’s thinking to mathematical cognition. On the other hand, if one is going to argue for something using an evolutionary framework, it behooves that person to supply a detailed model or story that will support it. Such an account seems lacking in Boyer's work. More to the point, the argument becomes even more problematic if it were to be applied to mathematical cognition (a claim, to repeat, that Boyer does not make). How many false conclusions, for example, has mathematical cognition produced, whatever definitions one wants to give of *true* and *false*?

For the moment, let us suppose that evolutionary theory will be able, eventually, to come up with a plausible explanation of our rationality, notwithstanding the arguments we have mentioned challenging that possibility. If so, any such theory that also attempts to promote a naturalistic world view would still run up against the arguments of Mark Steiner, author of *The Applicability of Mathematics as a Philosophical Problem*. Strictly speaking, Steiner's argument attempts to refute "non-anthropocentrism" rather than naturalism. But if Steiner is correct, the naturalist should not take comfort. For any form of naturalism, Steiner muses, is ipso facto non-anthropocentric, in that it would disallow a privileged status for humans in the scope of the universe. If, as Steiner argues, the success of mathematics can be shown to put humans in some sort of a privileged position, then naturalism has some problems to sort out. How does the success of mathematics put humans in a privileged position?

For Steiner, it is not so much the success of any one particular mathematical theory in an area of science. After all, there have been many failures of mathematics in addition to its successes. In this respect Steiner agrees with Hamming's third point, and is thus critical of Wigner's approach in citing specific success examples from physics while ignoring error stories. The use of *pi* by the statistician in Wigner's opening lines ignores all the failures in attempting to predict population trends. What Steiner is talking about is the success of mathematics as a grand strategy. It is a strategy that takes, for example, the raw formalisms of complex Hilbert space theory, and then boldly uses them as tools to make predictions about the quantum world, predictions that subsequently seem to be born out via experiment

How is this phenomenon anthropocentric? An analogy may be helpful here. Most cultures use a base ten numbering system. There is no universal agreement as to why this is the case, but the general consensus is that it has to do with our having ten fingers. (The Mayas, incidentally, used base twenty, and to many this confirms the "appendage hypothesis" for numerical base usage.) Now, what if successful theories of how the universe operates were based on multiples of ten? That would be anthropocentric to an extreme, as the only reason the number ten is special to us is due to how we appear to ourselves.

Suppose, further, that not only did the number ten have special significance, but time and time again other human aesthetic criteria also played a significant role in understanding the universe. Such occurrences, when looked at from a meta-level, would surely make one wonder why such privilege seems to fall on the human species. Yet this situation is precisely analogous to what mathematicians and scientists actually do when they rely on human notions of beauty and symmetry in the development of their theories.

In fact, such activity has been a longstanding and consistent strategy. Galileo, for example, pursued this tactic even though the best empirical evidence at the time did not support—indeed, it tended to disconfirm—his heliocentric theory.[2] He adopted it because it seemed much more elegant than the Ptolemaic model.[3] Most physicists generally admit that elegance, beauty, and symmetry hold primary sway in theory development. As Brian Green observes in *The Elegant Universe*, "Physicists, as we have discussed, tend to elevate symmetry principles to a place of prominence by putting them squarely on the pedestal of explanation."(Green, 1999, p. 374) G. H. Hardy argues that mathematics itself, at least what constitutes good mathematics, is driven primarily by aesthetic criteria such as economy of expression, depth, unexpectedness, inevitability, and seriousness, qualities that also seem to form standards for good poetry. (Hardy, 1967)

Two of these criteria, when taken together, appear paradoxical. In a beautiful mathematical theory, there is certainly the inevitable. A theorem marches on towards a conclusion that seems undeniable. But how can something inevitable also be unexpected? The answer lies in the proof of a theorem itself. A beautiful proof has, in its core, ideas that take the reader by surprise, almost like a series of brilliant moves in a chess match. And the surprises, when put in context, become stunningly beautiful. A good poem has that same effect. The pattern of words forms a symphony that contains many surprises to be sure, but when heard, seems paradoxically inevitable in that it had to be stated the way it was. For Hardy, the theories in mathematics that he deems "important" are precisely the ones that satisfy these aesthetic standards.

Steiner's book contains several examples of beautiful mathematical systems being used in applications to the physical world. His survey includes the use of complex analysis in fluid dynamics, relativistic field theory, thermodynamics, and quantum mechanics. Two of his examples are worth exploring in some detail.

First, consider Schrödinger's use of the wave equation. He begins with the equation

$$E = \frac{p^2}{2m} + V(x, y, z),$$

where he makes an assumption that the energy, $E$, is constant, so he can eliminate it by differentiating. After a series of formal manipulations he finally gets

$$i\hbar \frac{\partial \psi}{\partial t} = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(x, y, z) \right] \psi,$$

and then uses his result successfully in situations where energy is *not* constant. Some may wonder why the above mathematical symbols were included here, thinking that they are notational gimmicks only—designed to impress rather than educate, perhaps—and carry with them no special additional meaning. This is part of the point. To mathematicians, of course, the above symbols do carry meaning, but in manipulating them one is playing a game of sorts. As Steiner says, this is, "a perfect example of allowing the notation to lead us by the nose." (Steiner, 1998, p. 80)

The second example is well known among mathematicians and physicists: Maxwell's anticipation of a physical reality. In 1871 James Clerk Maxwell made a remarkable prediction resulting from his work in electromagnetic theory. He noted that the (experimentally confirmed) laws of Ampere, Coulomb, and Faraday, when put in their differential form, contradicted the law of conservation of electrical charge. How could this contradiction be resolved? Maxwell decided to tinker with the mathematical equation that represented Ampere's law. He eventually realized that if he added a term to it, the resulting equation would not only be consistent with the conservation of charge law, it would actually logically imply it. With no other warrant, Maxwell then boldly predicted that his new term would be found to correspond with some physical phenomenon. Maxwell died in 1879. Nine years later Heinrich Hertz demonstrated the reality corresponding to Maxwell's term—electromagnetic radiation.

Richard Carrier (M.Phil., Ancient History, Columbia University), is a freelance writer who is unimpressed by this episode, claiming that what Maxwell did is entirely consistent with naturalism. First, Carrier states that Maxwell's putting laws in differential form conforms to the naturalistic observation that nature works in continuous, not broken, processes. Second, Carrier argues that Maxwell took a logically sound hypothetical step: if charge isn't being conserved, then it must be going somewhere. Carrier then states,

> *Maxwell rightly picked the simplest imaginable solution first, which due to human limitation is always the best place to start an investigation, and which statistically is the most likely [as] simple patterns and behaviors happen far more often than complex ones. [Thus] Maxwell's moves [that] anticipated EM radiation [were] therefore a natural conclusion from entirely naturalistic assumptions.* (Carrier, 2003)

But with such language Carrier plays into Steiner's hands. Picking a *simple* solution in accordance with *human* limitations is precisely analogous to using the number ten as a means of unlocking secrets to the universe. It is quintessential anthropocentrism. Because of Carrier's background in history, one wonders if it is difficult for people who were not trained in science to appreciate how absolutely uncanny is the continued use of mathematical formalisms by physicists. Green, by contrast, seems to agree with Steiner's main point: at least unconsciously physicists have abandoned a raw naturalism in favor of a theory formation method that has principles of beauty embedded in its core. If they are correct, this approach certainly appears to be an anthropocentric—and by way of implication a non-naturalistic—strategy.

Or could it be naturalistic after all? Might it not be argued that plausible evolutionary models can be devised that would explain, for example, the human preference for symmetry? Such constructs seem possible, especially considering symmetries that might be adduced in examining our DNA code. But even if some model could be developed that would explain our preference for symmetry, how would a "blind chance" form of such thinking explain why such preferences are successful? After all, magical incantations are symmetrical, but they certainly do not work.

At least three strategies seem possible at this point. The first is to argue for some kind of probabilistic weighting mechanism that would drive physical processes towards the production of sentient life forms, and do so in such a way that their preferences for beauty coincide with the actual mechanisms of the universe. A second approach would involve an appeal to a primal basic position: it just so happens that the universe evolved in such a way that our notions for beauty work successfully in the development of theory formation. Finally, one might argue (along the lines of Wigner's second question) that what we call success came only because humans have invested a great deal of energy into science over the last 500 years. Who is to say that, if similar energies had been funneled in a different direction, there would be operating today a totally different paradigm, yet with the same degree of "success?" The success could be due to effort, not necessarily to some amazing connection humans have with reality. Thus, mathematical beauty poses no problems for naturalism whatsoever.

These are huge issues, and it would be presumptuous to think that a paper like this could settle them. Nevertheless, we can explore very quickly some tentative responses. First, with respect to the probabilistic weighting hypothesis, one might legitimately ask where the evidence is for this claimed weighting. As Keith Ward comments,

> *A physical weighting ought to be physically detectable, ... and it has certainly not been detected ... In this sense, a continuing causal activity of God seems the best explanation of the progress towards greater consciousness and intentionality that one sees in the actual course of the evolution of life on earth.* (Ward, 1996, p. 78)

Furthermore, if some kind of weighting could eventually be hypothesized and then tested, it may still be asked why such a weighting is biased in favor of humans. To a theist, there is no prima facie reason why God could not work with what appears to be "chance" (although the problem of determining what exactly one means by *chance* is by no means trivial). For a theist, the position humans seem to have of being able to understand the workings of the universe is, by itself, the result of the creative and purposive activity of God, even if our coming to be this way arose out of some kind of probabilistic weighting scheme.

Next, although primal basic explanations are needed at some level, invoking them in an effort to explain the apparent privileged status for humans in the universe—this is just the way it is and no more needs to be said—appears akin to pulling a rabbit out of a hat. A naturalist may choose precisely this option. But for a theist, again, the conviction that human notions of beauty relate successfully to our knowledge of how the universe operates reinforces all the more the belief that human creation is the result of the purposive activity of an intelligent being.

Finally, the idea that our constructs of success are ad hoc appears to be an objection without any realistic alternative. It is almost like saying, "Well, your theory makes sense, but only if one buys into some of your commonly accepted cultural notions. Other (unspecified) theories would be able to show that the success you claim is really arbitrary, and thus not privileged." Such a position, unfortunately, almost shuts down discussion. Of course, it is possible that other theories could be successful, but where are they? And why is it the case that the development of mathematics seems to be universal across cultures? It is noteworthy, for example, that the mathematics of pre-modern China, an independent and isolated culture, exhibits an impressive list of mathematical theorems also found in ancient Greece and other cultures, including the Pythagorean theorem, the binomial theorem, the solution of polynomial equations via Horner's method, and Gaussian elimination for the solution of systems of linear equations. (Howell and Bradley, 2001, Chapter 2)

I would suggest, in summary, that a theistic explanation is the best one in accounting for the continuing success of mathematical theories that ultimately grow out of aesthetic criteria. In assessing these arguments the reader is encouraged to adopt an approach similar to Reppert's in his defense of C. S. Lewis: there are, of course, valid points to be made on the side opposing these ideas, which should be looked at not as final answers, but as a catalyst to weigh various options. Human aesthetic values, and their subsequent use in successful physical theories, dovetail nicely with a Christian view that humans are created in the image of God. Whatever being in God's image exactly entails, it seems to include a rational capacity reflective of his that enables humans to understand and admire his creation. While not necessarily a final answer, such a perspective can be put confidently in the marketplace of ideas for appraisal, which is precisely what this paper has attempted to do.

## Endnotes

[1]An earlier version of this paper was first read at the bi-ennial meeting of ACMS, held at Huntington College, Indiana in June, 2005. It was subsequently published in the 2006 summer issue of *Christian Scholar's Review*, whose editors have given permission for this revision to appear in the *Journal of the ACMS*.

[2]For example, no stellar parallax shift could be detected from observed data, something that certainly would occur if the Earth revolved about the Sun. The problem, of course, is that stars are much further from the Earth than believed to be in Galileo's time, so the expected shift was not observed. In any case, detecting a shift would not have been possible with the technology then available.

[3]For a good treatment of Galileo, see "Galileo, the Church, and the Cosmos," in Lindberg, D. C. and Numbers, R. L., editors (2003), *When Science and Christianity Meet.* University of Chicago Press, Chicago, pp. 33-60 and 291-94.

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