Call for a Non-Euclidean, Post-Cantorian Theology

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1. Introduction

Theology, which may be defined as a systematic study of God and other doctrines of the Bible, has been studied for millennia, with volumes and volumes of literature that has been written over many centuries. Mathematics, which may be a bit harder to define[1], has also been studied for millennia, since the times of ancient civilizations, although the axiomatic treatment of mathematics seems to have begun in Greece some 2300 years ago.

Both of these disciplines seek to find timeless, universal truths about ideal things. They both start with certain self-evident assumptions (axioms, postulates, presuppositions, etc.) to draw conclusions by rational argument. For these reasons, it is perhaps not so surprising that theology and mathematics have influenced each other in the past, and it appears that they continue to cross their paths even today. One cannot deny that there is a significant amount of interplay here based on the integral relationship between the two disciplines. Moreover, the influence is mutual, or "bi-directional." Not only have faith and worldviews influenced the history and philosophy of mathematics, but mathematical concepts have also helped advance theology and understanding of the Scripture.

However, in this paper I shall only touch one aspect of these important ideas concerning the bi-directional relationship between mathematics and theology. The whole topic of bi-directional integration, which I call "strong integrability," will be dealt with in another paper.[2] The main point of the present paper is to propose a new approach to theology based on some discoveries in mathematics as well as their historical and philosophical implications on theology.[3]

In the following section, I shall mention three mathematical results that have had significant effects in philosophy and theology. They are non-Euclidean geometry, set theory, and Gödel's Incompleteness Theorem. All of these have been developed in the last 200 years, after most of the foundation of today's orthodox or reformed theology was established and solidified. I maintain that these recent discoveries in mathematics can and should make non-trivial contribution in the field of theology so that one can better understand certain biblical and theological concepts. Theologians should then revisit their discipline with the philosophical and logical implications of these mathematical ideas.

2. Three Mathematical Results of Philosophical Significance

The nineteenth century was undoubtedly one of the most important and fascinating periods in the history of mathematics. Carl Friedrich Gauss, the "Prince of Mathematicians," was proving new theorems in various branches of mathematics, especially in complex analysis, giving multiple proofs of the Fundamental Theorem of Algebra (which, despite its name, is really about complex analysis). Cauchy and Weierstrass laid a solid and firm foundation in analysis, something Newton and Leibniz had not done. Abel, Galois, and Cayley began a significant development in abstract algebra. In the first few decades of this mathematically busy century, one of the most incredible discoveries in mathematics was made, almost simultaneously by Bolyai (Hungary), Gauss (Germany), and Lobachevsky (Russia): non-Euclidean geometry.

(1) Birth of Non-Euclidean Geometry

Because of the enormous significance of this astonishing discovery, almost every book on the history of mathematics has a lengthy section explaining the history of the Parallel Postulate and the birth of non-Euclidean geometry. This section is based on a variety of sources, but a particularly well-written exposition is found in Marvin Jay Greenberg's book *Euclidean and Non-Euclidean Geometries: Development and History*. Here I present a rather detailed version of the birth of non-Euclidean geometry, for I believe it is essential to know this history before making applications in theological methods.

Euclid, who lived around 300 B.C., is best known for his book *The Elements*, a 13-volume masterpiece laying the foundations of geometry (and some number theory as well). He is called the "most widely read author in the history of mankind"(Greenberg, 1993, p. 9). In fact, no other books even come close except the Bible. *The Elements* was copied by hand for centuries, distributed throughout the civilized world, and translated into many languages. Since the first printed edition appeared in 1482, there have been more than 1000 editions of the book (Burton, 2003, p. 137). For over 2000 years, it was the standard textbook in geometry, and it is said that almost every educated person in the Western world studied Euclid's *Elements*.[4] Of course, one can criticize the level of logical rigor in this classic work if one compares it to today's standard: Bertrand Russell once said, "The value of Euclid's work as a masterpiece of logic has been very grossly exaggerated" (Greenberg, 1993, p. 70). Nevertheless, no one can dispute the crowning achievement of this ancient mathematician, who proved some 465 propositions based on only 23 definitions, five axioms, and five "common notions" (such as the transitivity of equality).[5]

One particular axiom, Axiom 5, was stated in a somewhat awkward fashion, compared to the first four. This is the Parallel Postulate of Euclid, sometimes affectionately called "Euclid's Fifth." It has been worded in a variety of ways and sometimes even replaced by one of the equivalent statements. Essentially, the axiom is equivalent to the following: given a line L and a point P not on L, there is one and only one line parallel to L through P. Interestingly enough, Euclid himself seemed a little skeptical about this axiom; he avoided using it until he absolutely had to—in his Proposition 29. For this reason, it has been suggested that Euclid may be the "first non-Euclidean geometer in history."[6]

If mathematics were just a string of symbols and a game to be played with the symbols, as formalists hold, there would not have been any problems. Mathematicians throughout the centuries, however, sought a more elegant axiomatic system: specifically, they wanted to eliminate the fifth axiom by proving it from the other axioms so that we would not have to put up with this awkward, less aesthetic axiom. The repeated efforts to do just that, by some of the most intelligent and competent mathematicians, is a story worth noting, not just in the context of mathematics but in the broader context of man's pursuit of truth.

Some of the first ones to attempt to prove the Parallel Postulate were Ptolemy (2nd century) and Proclus (5th century). They were followed by Simplicius (6th century), al-Jawhari (9th century), Nasir Eddin al-Tusi (13th century), and many others. Essentially all these men made the error of circular reasoning.[7]

After a few hundred years, mathematicians tried a new method. John Wallis (17th century), for instance, proposed a new axiom (existence of similar triangles) to prove Euclid's

Fifth. He succeeded doing this, but it turned out that his new axiom is logically equivalent to the Parallel Postulate. At the beginning of the 18th century, Girolamo Saccheri, a Jesuit priest, used what is known today as "Saccheri quadrilaterals" to produce a contradiction, which he needed to prove the Parallel Postulate. He would have succeeded in reaching a contradiction if he could show that the top angles of Saccheri quadrilaterals cannot be acute. He could not. He is then quoted as saying, "The hypothesis of the acute angle is absolutely false, being repugnant to the nature of a straight line!"(Burton, 2003, p. 529). It is interesting to observe that the word "repugnant" was used when what seems obvious could not be rigorously proved. I shall come back to this point later.

About the same time, Alexis Clairaut used the existence of a rectangle to prove the Parallel Postulate. After all, who could deny the existence of a rectangle when we sleep on rectangular beds in a rectangular room in a rectangular house built on a rectangular piece of land? But this was no success at all, for, later the existence of a rectangle was shown to be equivalent to the Parallel Postulate. Shortly after, Johann Lambert[8] used a method similar to the one considered earlier by Saccheri. His conclusion was also similar to that of Saccheri: when he could not prove the Parallel Postulate, he said it must be true because, otherwise, it "would result in countless inconveniences" (Greenberg, 1993, p. 160). Adrien-Marie Legendre and F. A. Taurinus, in early 19th century, also published several attempts to establish the Parallel Postulate on the basis of the other axioms.

There are several points to be made concerning the efforts of all these mathematicians. First, note that they all believed that which they were trying to prove—in other words, there seems to have been a preconceived notion. But who could blame them? After all, Euclid was the standard that had already been around for 2000 years (talk about standing the test of time!). Observe also that all of these efforts showed that, if Euclid's Fifth is false, it would lead to ridiculous conclusions, such as the impossibility of rectangles, impossibility of having similar and non-congruent triangles, and a pentagon with five right angles. Some of them, at this point, used words like "repugnant" and "inconvenient" because they could not possibly accept such a counter-intuitive geometry. However, at the same time, they were honest enough, as mathematicians, to avoid taking a dogmatic stand. They sincerely pursued truth through a purely logical and mathematical method and admitted—directly or indirectly—that they had not succeeded.[9] These are great lessons, applicable in all forms of inquiry and academic discipline. I shall return to these philosophical and epistemological implications later, particularly in the context of theology.

The frustration level in the mathematical community was rising. In 1763, G.S. Klügel wrote a paper on how *not* to prove the Parallel Postulate. The problem was called the "scandal of geometry" (Greenberg, 1993, p. 161). In fact, one of the most fascinating stories in this area is how Wolfgang Bolyai, who worked for many years on the Parallel Postulate, told his son John to stay away from this particular problem, writing in a letter:

> *I have traversed this bottomless night, which extinguished all light and joy of my life. I entreat you, leave the science of parallels alone. . . . I saw that no man can reach the bottom of the night. . . . I have traveled past all reefs on this infernal Dead Sea and have always come back with broken mast and torn sail. . . . I thoughtlessly risked my life and happiness*.[10]

These are rather depressing words, coming from a person who devoted his life to find the truth about Euclid's system.

Now, what does a typical son do when his father tells him *not* to do something because it is too hard? Of course, the son John decided to do just that—and, doing so, he became one of the most important names in the history of mathematics! With a completely new idea, perhaps with a less preconceived notion and a little more willingness to accept new things, John Bolyai of Hungary wrote in 1823 to his father Wolfgang: "I have discovered such wonderful things that I was amazed…. Out of nothing I have created a strange new universe"(Greenberg, 1993, p. 163).[11] Non-Euclidean geometry was thus born.[12]

In January, 1832, Wolfgang Bolyai, a close friend[13] and a former classmate of Gauss, sent a copy of his book *Tentamen*, which included (in an appendix[14]) non-Euclidean geometry developed by his son, to this most respected mathematician of the day. Interestingly, Gauss replied with the now-famous words: "To praise it would amount to praising myself," adding that Gauss himself had been developing these ideas for 35 years. Even more interesting is the fact that Gauss, in spite of his unchallenged reputation in the mathematical world, was unwilling to make his controversial discoveries public and was planning to publish his work on non-Euclidean geometry after his death. He was afraid that the followers of Immanuel Kant would attack his new geometry viciously. In a letter to Friedrich Bessel, an astronomer, Gauss wrote the following (Burton, 2003, p. 546):

> *I fear the scream of the Boeotians [a figurative reference to the dullards, for the Boeotians were reputed to have been one of the most simple-minded Greek tribes] were I to completely express my views.*

While Gauss and Bolyai were corresponding with each other concerning this breakthrough in the history of mathematics and perhaps philosophy in general, they were totally unaware that an unknown young Russian mathematician had worked tirelessly on the same problem and would become the first to put non-Euclidean ("imaginary" in his term) geometry in print (in a monthly journal published by Kazan University) in 1829: his name is Nikolai Lobachevsky. He was criticized by numerous Russian scholars for his "insolence and shamelessness of false new inventions," and his subsequent papers were rejected by academic journals. No mathematician in his day gave him public approval. Even Gauss, who recognized the masterful accomplishment of Lobachevsky and who in his old age learned Russian so that he may read Lobachevsky's other works in the original language, did not endorse the work publicly. Later, Lobachevsky was fired from the university after twenty years of service there; his son died suddenly, and he himself became blind. The tragic life is similar to that of John Bolyai, who suffered mental depression, partly because of Gauss's response and partly because his *Appendix* was almost completely ignored by the mathematical community.[15]

Thus, these two unknown mathematicians "dethroned" the once-irrefutable idol called Euclidean geometry, which had reigned as the ultimate and absolute certainty for more than 2000 years. Concerning these two young but incredible mathematicians with truly amazing talent, Burton says the following (2003, p. 554):

> *[They] were denied the welcome that so many centuries of anticipation seemed to promise. . . . Both men were aware of the revolutionary magnitude of their*

geometrical theories and expected to be rightfully heralded; both were bitterly disappointed with the response.

Incidentally, the story of these men somewhat resembles the incarnation of Christ. The Messiah had been anticipated for 2000 years to come to the world, free His people from bondage, and teach the truth. But when the time came, God the Son came as a son of a poor family and grew up in an obscure town of Nazareth. The established authority relentlessly attacked Him. Their preconceived notion and blind faith in the then-accepted system of religion did not allow them to see the truth. Among the educated, only Nicodemus appears to have noticed the significance of Jesus, but even he did not show his support publicly until the burial of Jesus. Perhaps it is no accident that mathematician-historian E. T. Bell calls Lobachevsky "the great emancipator" (Greenberg, 1993, p. 184).

Just as Gauss had predicted, many of Europe's intellectuals rejected this new geometry as non-sense. The academic world at that time was heavily influenced by the philosophy of Immanuel Kant, who held that Euclidean geometry is "inherent in the structure of our mind" (Greenberg, 1993, p.182).[16] Just as the theology of the Pharisees was the established conservative creed in the days of the New Testament, Euclidean geometry was the ultimate truth for over two millennia. There were many who still held the view that replacing the Parallel Postulate with the Hyperbolic Parallel Postulate[17] would eventually produce a contradiction. After all, even though the pioneers of non-Euclidean geometry were confident that there would be no contradictions, none of them could actually *prove* that their set of axioms was consistent (the axioms *could not* produce any contradictions).[18]

The final blow to Kant's school of philosophy and skeptics of non-Euclidean geometry came in 1868, when Eugenio Beltrami created a model of non-Euclidean (hyperbolic) geometry using Euclidean geometry. This was absolutely astounding, for it basically implies that if Euclidean geometry is consistent, then so is non-Euclidean geometry. It is the concept of "relative consistency." This is a true irony. I mentioned earlier in a footnote that Saccheri made serious efforts to clear Euclid's name by demonstrating that his Fifth Postulate is a necessary consequence of the other postulates of Euclidean geometry. What would have happened if Saccheri had succeeded? Here is what the logic teaches us in that case:

- Based on the other Euclidean axioms, Euclid's Parallel Postulate would be true.
- Hence, if Euclidean geometry is consistent, any model created by it must be valid.
- Beltrami used Euclidean geometry to create a model of hyperbolic geometry.
- Hence, hyperbolic geometry would be valid.
- But hyperbolic geometry clearly contradicts the Parallel Postulate and thus would be invalid.
- Therefore, Euclidean geometry would be inconsistent.

The irony is that, if Saccheri or anyone else had succeeded in proving the Parallel Postulate to establish dogmatically that Euclidean geometry is the only possible geometry, it would have totally devastated Euclidean geometry, proving it to be inconsistent! Little did they know that if they had successfully built the tower, they would have completely demolished it. This, I say, is a truly amazing story. Burton summarizes it quite well (2003, p. 529):

Indeed, had Saccheri actually accomplished his purpose and proved the Parallel Postulate from the remaining axioms of Euclidean geometry, he would not have vindicated Euclid. Quite the contrary, he would have dealt a terrible blow to Euclid. Euclid was vindicated by the discovery of non-Euclidean geometry, for its existence demonstrates that the Parallel Postulate is independent of Euclid's other axioms, so that it truly widens the axiomatic base on which Euclid's geometry stands. We must admire the Great Geometer all the more; the introduction of the Fifth Postulate, so decidedly unaxiomatic in appearance, yet an independent postulate, was a stroke of pure genius.

What then is the conclusion of the whole matter? Is Euclidean geometry true? It is the wrong question to ask. It makes as much sense as asking, "Is the binary system false and the decimal system true?" It is a matter of taste, or of convenience. For an architect and a carpenter, Euclidean geometry is practical. (We all appreciate living in houses where the floor and ceiling are horizontal.) For an astronomer or a physicist, however, Euclidean geometry may not be capable of describing the universe. Einstein certainly found non-Euclidean geometry extremely useful. For someone navigating around our spherical globe, yet another version of non-Euclidean geometry (called spherical geometry), where there are no parallel lines, is helpful. The point, then, is *not* to ask which geometry is true—and refrain from being dogmatic simply on account of one's "intuition." As for Kant, we now know that he was severely limited by his observation and intuition, not unlike a person who believes in a "flat (Euclidean) earth" today. Because the earth is spherical and the universe may also be non-Euclidean, Wallace and West call this irony "perfect retribution" since Kant made his statement about the absolute nature of Euclidean geometry while "living on, if not in, a non-Euclidean world" (2004, p. 414).

There are many things we can learn from the long history of this "Euclidean struggle." Appealing to "obvious" facts and dismissing "repugnant" results do not always lead to truth. This, however, is often the case in many academic disciplines, including theology. Argument based on the "inherent" nature of man's mind is equally invalid, even if it is made by a leading philosopher. Intuitive absurdity, such as the non-existence of a rectangle, is not a good reason for dismissing an idea.[19] Sometimes common sense is the chief enemy in man's pursuit of truth. I shall discuss these and other philosophical implications in the next section, after I complete this section with two more historically and mathematically astonishing results.

(2) Struggle with Set-Theoretical Questions

About the time Lobachevsky was abruptly dismissed from his position at Kazan University in Russia, a child was born in St. Petersburg, Russia, who would ultimately revolutionize the entire landscape of mathematics. His name is Georg Cantor. During the last few decades of the 19^{th} century, he pioneered a new branch of mathematics—theory of sets—which was to become a major foundational theory, influencing most, if not all, branches of mathematics. His earlier work in analysis led him to study certain subsets of real numbers, which in turn motivated his career-long obsession with transfinite numbers ("actual infinity"). In spite of the harsh criticism by many of his peers, he almost single-handedly developed the theory of these transfinite cardinals, boldly stepping into the realm of the infinite, then considered to be a forbidden territory by philosophers, theologians, and mathematicians alike. He went on to prove some remarkable results, which historian Burton calls the "most disturbingly original contributions to mathematics in 2500 years" (2003, p. 625). They include theorems which seemed very odd in his days, including the following:

- There are as many positive integers as squares of positive integers (Galileo's paradox).
- There are as many rational numbers as positive integers.
- There are as many algebraic numbers as positive integers.
- However, the set of irrational numbers (in fact, even the smaller set of transcendental numbers) is a whole lot bigger than the set of positive integers (he proved this by the famous "diagonalization" argument).
- In fact, there are infinitely many "sizes" (cardinal numbers) of infinite sets.
- The set of points in a square is just as big as the set of numbers in the unit interval.

This last theorem was particularly counter-intuitive; the proof of this result is what Cantor was referring to in the now-famous letter to his good friend Dedekind, dated June 29, 1877, "*Je le vois, mais je ne le crois pas* (I see it, but I do not believe it)" (Aczel, 2000, p. 119).[20] It was a decade before Beltrami was to establish the relative consistency of Euclidean and non-Euclidean geometries.

As expected, this unorthodox theory encountered much contempt. Non-Euclidean geometry had also been attacked by skeptics a few decades earlier, but most of the accusers were, at least, non-mathematicians. In this case, however, fierce attacks came from fellow mathematicians—in particular, Cantor's former professor Leopold Kronecker. Many of these paradoxical results about infinite sets were considered "absurd" by many contemporaries (note that this same word appeared before in attacks of non-Euclidean geometry) and thus were rejected. In fact, Burton has an interesting comparison (2003, p. 628):

> *Some mathematicians of the day could accept, albeit reluctantly, Cantor's infinite sets, taking an attitude that has been compared with that of a gentleman toward adultery: better to commit the act than utter the word in the presence of a lady. It was an actually infinite number that was forbidden, and its use forced Cantor to live the rest of his life within a storm.*

Often this type of mathematical unorthodoxy is compared to theological heresy. Kronecker accused Cantor of being a "corrupter of youth" for teaching transfinite numbers (Aczel, 2000, p. 132). Again, Burton describes the attitude of Cantor's contemporaries vividly (2003, p. 629):

> *Cantor became a heretic. The outcry was immediate, furious, and extended. Cantor was accused of encroaching on the domain of philosophers and of violating the principles of religion. Yet, in this bitter controversy, he had the support of certain colleagues, most notably Dedekind, Weierstrass, and Hilbert. Hilbert was later to refer to Cantor's work as "the finest product of mathematical genius and one of the supreme achievements of purely intellectual human activity."*

Kronecker, whom Burton calls "the focus of Cantor's troubles, a sort of personal devil" (2003, p. 629), also objected to new ideas in analysis formulated by Dedekind and Weierstrass, accusing them of using "quantities whose being was merely established by the free use of 'theological existence proofs'" (Burton, 2003, p. 631). Again, it is interesting to note that mathematical arguments were compared with theological ones.

While Cantor was called a heretic and an evil man, Cantor's view on himself and his mathematics was quite contrary. A deeply religious man, he believed that infinity was God-given. He constructed infinite levels of infinity by proving that the power set of any set (including infinite sets) is strictly larger than the set itself, and he believed that the ultimate level of infinity (which he called the "Absolute") was God Himself (Aczel, 2000, p. 132). In addition, Cantor maintained that his theory and discoveries would eventually make significant contributions not only in mathematics but in philosophy and theology also. Indeed, today, some philosophers and theologians are looking at age-old theological questions in light of Cantor's theory of infinity and set theory. However, much more can be gained by looking at theology in a post-Cantorian way (see the next section). In particular, various theorems of infinite cardinals and infinite sets could shed light on the study of God, or theology proper.

Cantor was by no means the first one to consider the link between God and the mathematics of the infinite. In fact, the concept goes way back to, at least, Augustine, who held that actual infinity resides in the mind of God. In *City of God*, he writes that "infinity of numbers is not outside the comprehension of him [God] . . . every infinity is, in a way we cannot express, made finite to God" (Byl, pp. 258—259). This idea, which he called "a way we cannot express," may be exactly what Cantor did many centuries later.

Cantor continued to work hard on establishing his set theory in the midst of the fierce opposition referred to earlier. Even his nervous breakdown and other health problems did not stop him in his quest for mathematical truths. However, there rose a more serious problem: his foundational ideas of sets had very serious logical flaws, which appeared in various paradoxes. One paradox, bearing Cantor's name, was due to the set of all sets, which led to a contradiction. Another, more famous, paradox was due to Russell, published in 1903 (only a century ago!). It has to do with a self-referential statement.[21]

Because of these flaws coming from too much generality, Cantor's initial attempt to axiomatize set theory is now called the "naïve set theory." Others even called it a "set theory built on the sand." Later, set theorists such as Zermelo, Fraenkel, von Neumann, and Peano would fix these problems and eventually come up with a well-formulated set of axioms, known today as the "ZFC axioms." (This is the "set theory built on the rock.") The "C" here stands for the Axiom of Choice, which turned out to be an extremely significant axiom, as we shall see shortly.

It may be surprising that, in this axiomatic system, the most basic entity is the empty set, denoted either by ø or { }, which is defined to be the number 0. Then, 1 is the set whose only element is { }, or 1 = {{ }}. Likewise, 2 = {0, 1} = { { }, {{ }}}. In this fashion, the set ***N*** of positive integers is constructed, all out of the empty set. Based on ***N***, one can construct the integers, rational numbers (using ordered pairs), real numbers (using Dedekind's cuts or Cauchy sequences), and complex numbers. There is no wonder someone called mathematics "nothing more than a bunch of brackets."[22] All of our number systems are then based on the single entity called the empty set (which is not exactly "nothing"). Some even remarked that set theorists (Peano in particular) created everything out of nothing!

When these pioneers of axiomatic set theory were formulating the axioms at the beginning of the twentieth century, two questions came up which puzzled them, including Cantor himself. One was the conjecture that there is no set that is strictly bigger than the integers but strictly smaller than the set of real numbers. This conjecture became known as the Continuum Hypothesis (CH). When Hilbert presented his famous list of 23 open problems for the twentieth century in Paris in 1900, CH was the first question listed. Cantor firmly believed this, yet all of his efforts to prove it led to no success. Meanwhile, at the Third International Congress of Mathematicians (Heidelberg, 1904), a Hungarian mathematician Jules C. König presented a paper refuting CH. Extremely defensive and mentally unstable, Cantor became more dogmatic about CH. In fact, Aczel describes this almost religious "conviction" of Cantor as follows (2000, p. 165):

> *It was the word of God. Cantor described the continuum hypothesis to colleagues in exactly these words, and he even believed that God would protect the continuum hypothesis from its attackers.*

Cantor was somewhat vindicated; within a day, Zermelo found a fatal flaw in the proof of König. Subsequently, Cantor worked even harder to prove CH before Zermelo could fix König’s proof and disprove CH. By this time, Cantor saw himself as a "faithful secretary to God," given the task of "recording God's words to the world" (Aczel, 2000, p. 177). He never succeeded. The irony—an extremely sad one—is that he (or anyone else for that matter) *could not have possibly proved it or disproved it!*

The other puzzling problem was the Axiom of Choice (AC), referred to earlier. This states that, given a collection of infinitely many disjoint non-empty sets, a set can be constructed in which exactly one element comes from each set in the (infinite) collection. It is an intuitively obvious, almost self-evident proposition, just as the Parallel Postulate is. Originally, AC was used to prove a result called the Well-Ordering Theorem by Zermelo in 1904.[23] Controversy erupted (again). The construction requires a way to pick infinitely many things, one at a time, and many reputable mathematicians were not willing to accept that it could be done. Some progress was made for the next few decades: an important theorem in analysis called the Hahn-Banach Theorem was proved using what is known as Zorn's Lemma, which turns out to be equivalent to AC. Cantor considered this problem from his theological viewpoint, also; he believed that God can order any collection of things, so the Well-Ordering Theorem must be true with Him. Hence, Cantor thought, AC must be true as this "poses no problem for an all-knowing, all-powerful God" (Byl, 2004, p. 275).

The story so far is very much like the agonizingly long struggle with Euclid's Parallel Postulate. Many people tried to prove it and failed. Indeed, the conclusion was also quite similar. In 1940, Kurt Gödel produced a model of set theory (i.e., a system that satisfies all the Zermelo-Fraenkel axioms) in which both AC and CH are true. Hence, any efforts to disprove AC or CH had been in vain. Inconsistency of these two propositions with other axioms of set theory was ruled out. Then, even more shocking news spread through the mathematical community: in 1963, Paul Cohen[24] produced models of set theory in which either AC, CH, or both are false. (Note that this result is only 40 years old.) In other words, CH, the problem with which Cantor struggled so much in the last years of his life (and which he thought was the word of God), was totally independent of the other axioms of set theory. Whether or not to accept it is a matter of taste. The same is true with AC. These two propositions are, therefore, modern counterparts to Euclid's Parallel Postulate, which took mathematicians over 2000 years to fully understand. Cohen's astonishing paper (jointly written with R. Hersh) was entitled "Non-Cantorian Set Theory," which, of course, is analogous to the term "non-Euclidean geometry." Cantor would probably be happy to hear such a term: he believed in mathematical liberty—he left the famous words, "The essence of mathematics lies in its freedom."

Once again, the mathematical drama had a very surprising end. The truth is not always easy to obtain. An amazing thing is that mathematicians actually proved that certain mathematical statements can be neither proved nor disproved, and they did so using mathematics.

(3) Gödel's Incompleteness Theorem

The last of the three philosophically significant results in recent mathematics is also due to Kurt Gödel. Perhaps his Incompleteness Theorem is most crucial as far as its epistemological and mathematical implications are concerned.

In light of the shocking discovery concerning Euclid's Parallel Postulate and the failure of naïve set theory, together with tremendous progress made in logic, axiomatization was thought of as the key to achieving rigor in every branch of mathematics at the beginning of the 1900s. This was a way to build the house on the rock instead of the sand. The basic idea was to find, for every branch of mathematics, a reasonable set of axioms such that it can be proved consistent (no contradictions would ever arise) and complete (every statement can be either proved or disproved). This ambitious undertaking was led by David Hilbert and was called "Hilbert's Programme." Perhaps this was partially motivated by the humanistic and naturalistic worldview of the day—to eliminate from mathematics any necessity for, or faith in, God. Whatever the motivation was, Hilbert aggressively advanced his program. His philosophy of mathematics, known as formalism, is expressed in his own words, "Mathematics is a game played according to certain rules with meaningless marks on paper" (Burton, 2003, p. 581). Paul Gordan attacked this view, saying, "This is no longer mathematics, it is theology" (Burton, 2003, p. 581).[25]

Hilbert, in an attempt to prove the consistency of Euclidean geometry, constructed a purely arithmetic model of Euclidean geometry. Thus, Euclidean geometry is shown to be consistent as long as arithmetic (of the reals) is. Of course, today we see that this is just moving the same problem to another branch of mathematics, but it turned out to be an important accomplishment as others were trying hard to prove the consistency of arithmetic. Things were going so well for a while that Hilbert stated in 1927, "I believe that I can attain this goal completely with my proof theory. . . Mathematics is a presupposition-less science."[26] Three years later, in 1930, upon his retirement from Königsberg, Hilbert concluded his final speech with the famous words of optimism(these words now appear on his tomb in Göttingen): "*Wir müssen wissen, wir warden wissen* (We must know; we shall know)."

Mathematical dramas are sometimes painfully unpredictable. Just a year later, Gödel proved one of the most revolutionary results in all of mathematics: his Incompleteness Theorem, and this definitively ended the dream of Hilbert and his formalist colleagues. More precisely, this genius in his mid-20s proved the following two propositions:

- In any system of axioms large enough to include arithmetic, there are statements that can neither be proved nor disproved (undecidable sentences). In other words, every mathematical system is necessarily incomplete because it contains undecidable statements expressible within the system.
- Worse yet, the consistency of the system itself is one of these undecidable statements. In other words, no system of axioms can be proved consistent within the system itself.

What do these results mean? For one thing, Hilbert's dream of inventing a complete system in every branch of mathematics turned out to be just that—a dream. The reality is that he could not have done that for *any* branch of mathematics. Furthermore, his other dream of proving the consistency of such branches as Euclidean geometry and set theory also turned out to be impossible. Burton calls the Incompleteness Theorem a "milestone in the history of modern logic" and a "mortal blow to the final objective of Hilbert's career, a formalized version of all classical mathematics" (2003, pp. 581—582). Thus, out of the 23 problems Hilbert proposed in 1900, the first one (CH) turned out to be impossible, and so did the second one (to show the consistency of the axioms of arithmetic).

The philosophical implications of the Incompleteness Theorem are extensive. Mathematics had been considered the ultimate, absolute truth for centuries. It was bad enough that the certainty of Euclidean geometry was challenged and overthrown. But now, it was proved (using mathematics) that the consistency of any axiomatic system of mathematics is mathematically unprovable. In Greenberg's words, Gödel "provided a formal demonstration of the inadequacy of formal demonstrations!" (1993, p. 298). Byl calls this a "devastating effect" (2004, p. 142) in the philosophy of mathematics; he then quotes several authors to underscore the importance of this theorem (2004, pp. 142—143):

> *"Although I believe in my heart that mathematics is consistent, I know in my brain that I will not be able to prove that fact, unless I am wrong. For if I am wrong, mathematics is inconsistent. And if mathematics is inconsistent, then it can prove anything, including the statement which says that mathematics is consistent" (logician Christopher Leary).*

> *"Gödel has taught us that not only is mathematics a religion but it is the only religion able to prove itself to be one*" *(John Barrow).*

> *"The splendid certainty which I had always hoped to find in mathematics was lost in a bewildering maze*" *(Bertrand Russell).*

Greenberg quotes another well-known mathematician, René Thom, on the implications of Gödel's Theorem (1993, p. 299):

> *Mathematicians have only an incomplete and fragmentary vision of this world of Ideas. . . , we have to recreate it in our consciousness by a ceaseless and permanent reconstruction. . . and the ultima ratio of our faith in the truth of a theorem resides in our intuition.*

We have now looked, rather extensively, at three recent mathematical ideas of enormous magnitude and consequence. Each is surprising in its own unique way, and each is a drama, complete (no pun intended) with a colorful cast of personalities, human struggle, triumph and defeat. All the characters had the same goal—to find mathematical truths. Many of them had sincerely believed certain things, which turned out to be false or, perhaps worse yet, unknowable. What can one learn from these scenes from the rich history of mathematics, particularly in the context of finding theological truths? I shall move onto that discussion now.

3. Proposal for a Non-Euclidean, Post-Cantorian Theology

By the term "non-Euclidean, post-Cantorian theology," I mean an approach to theology based on and in light of the epistemological implications as well as contents of non-Euclidean geometry and the development of set theory and of foundations, including Gödel's Incompleteness Theorem. Let us briefly summarize these implications, or "lessons from history."

The first obvious lesson is one that I have mentioned several times already: what appears to be "absurd" may not be absurd. The reference here is obviously to the Parallel Postulate, the negation of which was often dismissed as "repugnant" or "absurd." With the discovery of various non-Euclidean geometries, we know now that this is not absurd at all in the world of abstract mathematics. In fact, the Hyperbolic Parallel Postulate may actually be more "true" and "real" than the Euclidean one as the theory of relativity implies. The existence of a rectangle, for instance, was considered "absurd" to doubt, and in Cantor's time, it was "absurd" to conclude that an interval, as a set, is as large as the unit square. Indeed, this and other theorems involving infinity were so hard to believe that even professional mathematicians of the day rejected the entire theory that Cantor was building (i.e., the "paradise Cantor has created," according to Hilbert). An appeal to "absurdity," therefore, must not be used as a proof in any argument, whether in mathematics or theology. Similarly, an appeal to "self-evident" truths should be scrutinized carefully. When one revisits theology with this in mind, it may be surprising to see how many arguments depend on phrases like "obviously absurd" and "clearly a contradiction."

Another lesson is similar to the first one: hidden assumptions must be carefully examined. Because we are human beings, we approach problems with bias. Albert Einstein once said that common sense is "nothing more than layers of preconceived notions stored in our memories and emotions" (Bartusiak, 2000, p. 38). We shall present some examples of assumptions hidden under theological arguments. When proving some proposition, especially by contradiction (indirect proof), it is extremely important to show precisely where the contradiction occurs—in the form "*p* and not *p*."

In addition, in light of Gödel's Incompleteness Theorem, one must accept the fact that there may be certain propositions that are undecidable. In systematic and biblical theology, the set of axioms is the biblical texts. If one accepts the infallibility of the Scripture, as most evangelical Christians hold, the original texts are the axioms, from which one is to draw theological and practical conclusions. However, we all know that the Bible addresses neither every practical issue that we face nor every theological issue we ponder. Hence, these things on which the Bible is silent should perhaps be considered undecidable propositions in this axiomatic system.

Finally, related to the last point, one must be willing to accept that there may be more than one "correct" answer that is consistent with the axioms. Just as there are basically three geometries (with negative, zero, and positive curvatures) on the plane, each of which is consistent with the four other axioms of Euclid, various different models could exist within the axiomatic system of the Scripture. When this happens, one should refrain from taking dogmatic positions. Being dogmatic on these issues would be just as wrong and unintelligent as believing only in Euclidean geometry or in the negation of the Continuum Hypothesis.

Now, before looking at specific arguments in theology as examples, let me just say that I am fully aware of various schools of thought concerning the use of logic and mathematical methods in theology. Some do not believe in it. I shall address that issue in Section 4, where, among other things, I defend the use of classical logic, including the law of excluded middle (which neither intuitionist mathematicians nor some theologians accept). For now, we turn to some examples of how these historical events and theorems of mathematics could shed light on theological arguments. I will focus my attention on four topics of theology that are of particular significance: the existence of evil, sovereignty and free will, hypostatic union, and the Trinity. All of these topics have something in common: theologians and non-theologians agree that most (if not all) human illustrations fail to explain these difficulties and that these concepts are impossible to fully comprehend (although there is no proof of the impossibility). They seem to admit, willingly and quickly, that these are beyond our finite intelligence. *But this is exactly the main point of this paper*: I am proposing that these theological ideas be revisited with the lessons we have learned from the history of mathematics—a history of man's pursuit for absolute truths. Hence, instead of simply giving up and saying, "Well, that's beyond us," let us consider if a post-Cantorian, non-Euclidean approach to theology could shed light to these century-old theological problems.

(1) Existence of Evil

One of the difficult questions—both philosophical and theological—is the presence of evil. The problem—often called a paradox—goes as follows:

- Evil does exist (the innocent suffer).
- Hence, if God is willing to prevent evil, He is not able (hence not omnipotent).
- If God is able to prevent evil, He is not willing (hence not good).

Hence, the crux of the argument is that the presence of evil (which hardly anyone denies) makes it impossible for God to be both good and omnipotent. In other words, either the goodness or the greatness of God is in jeopardy if evil exists. The "evil" referred to in this problem is two-fold: the natural type, including natural disasters such as earthquakes, tornadoes, etc., and the moral type, including criminal acts such as murder of the innocent, abuse, etc.

These three components of the problem—evil, goodness, and greatness—naturally lead to the three approaches made by various theologians and philosophers.[27] Finitism, held by Edgar S. Brightman, etc., basically denies his greatness, maintaining that God must work with free human will and thus is unable to stop certain things from happening. Others, such as Gordon H. Clark, a Calvinist, redefine the "goodness of God," implicitly reducing His goodness. Clark, for instance, states that God is "the *ultimate* cause of sin, not the immediate cause of it" (Erickson, 1983, p. 418). Yet others, such as Mary Baker Eddy, the founder of Christian Science, deny the existence of evil altogether, dismissing the concept as a human illusion. To them, even death is not real but illusory.[28]

Let us examine the basic logical structure of the propositions involved here. Is the logic tight? Where is the "contradiction" of the paradox? Note the mathematical form of the argument. The "proof" goes something like this: "By the given, evil exists. Now, suppose God is willing to prevent it. If God is also able to prevent it, then evil would not exist. Hence, in this case, God is not able. Suppose, on the other hand, God is not willing to prevent it. In this case, God is not good. Hence, God cannot be both able (omnipotent) and good."

Where is the flaw in this argument? What are the underlying assumptions? It requires careful checking to answer these questions, very much in a manner similar to how Gauss examined various attempts made by others to prove the Parallel Postulate. One assumption implicit in this argument is the proposition "If X is willing to and able to do Y, then X is going to do Y." Here, X is God and Y is the act of preventing evil. Perhaps it is this presupposition that one needs to examine. Do any of the axioms (biblical texts) support this? In other words, can one get this proposition from the Scripture anywhere? Or is it one of those "self-evident truths," the negation of which is "absurd" or "repugnant" to most of us? Is it possible that we do not see the "whole picture" as God does? For example, is it possible that X is indeed perfectly willing to and able to do Y but refrains from doing so due to other factors that are fully consistent with His attributes (such as His mercy to those causing the evil)? Whereas these questions may not solve the problem completely, it may be worthwhile to consider them. This type of abstraction in theology, using letters like X and Y, could be termed "algebraic theology": it is a nice way to extract various hidden assumptions, make the argument more precise, and detect exactly where the contradiction may be occurring.[29] It can be applied in a wide variety of problems and problem passages.

(2) Sovereignty vs. Free Will

This well-known, thoroughly debated, and historically significant problem has split the theological community into two schools: those stressing the sovereignty and unconditional election of God (Calvinists) and those who stress the free will of man (Arminians). The problem is often phrased in the form of a question toward Calvinism: how can one be held accountable for unbelief if he *cannot* have faith by God's sovereign choice? It seems clear from the Bible that

- God sovereignly chose the "elect" before the foundation of the world.
- The non-elect will not and cannot have soul-saving faith.
- They are accountable for rejecting the truth.

Again, let us use the language of algebra to extract the essence of the argument. The question is the following: "if one is incapable of doing X, can he be held responsible for not doing X?" At this point, some theological books go into real-life analogies such as innocence by reason of insanity and a little child not being able to read a warning sign. But what makes us believe that inability excuses us from our responsibilities? Note that I am not arguing for or against this proposition. The point is that this statement may be true in many human and legal applications in our society while it is not by any means a logical necessity. What may be "obvious" in one sphere may not be so "obvious" or evident in another. There is a hidden assumption (responsibility implies ability), which must be carefully investigated.

A related topic is the incompatibility of the implications of Calvinism and Arminianism. On the one hand, Calvinists argue, based on the Scripture, that election is unconditional and that grace is irresistible. On the other hand, Arminians argue, also based on the Scripture, that man seems to have a choice of believing or not believing. Both of these schools try hard to be, and claim to be, consistent. Clearly, however, one cannot hold both of these positions and say that election is both conditional and unconditional (or that grace is resistible and irresistible).

Is it possible that these two "systems" are equally valid, both satisfying the axioms (the Scripture) and free of contradictions, just as the Euclidean and Hyperbolic Parallel Postulates are both valid although one negates the other? Is it possible that what Calvinists call "absurd" and "repugnant" in the Arminian position (and what Arminians call "absurd" and "repugnant" in the Calvinist position) is neither absurd nor repugnant in a sort of "neutral"[30] theology? These are some of the questions that need to be studied in depth.

(3) Hypostatic Union

One of the most difficult questions to answer is the problem of hypostatic union: Jesus is both fully God and fully man. This is a problem involving the finite and the infinite, both the divine and the non-divine, both the agent and the object of creation. God is everywhere, but man occupies one place at a time. God knows all things, but man does not. God can do all things, but man cannot. The problem at hand is a problem of two logically contradictory natures. It is said that "this is one of the most difficult of all theological problems, ranking with the Trinity and the seeming paradox of human free will and divine sovereignty" (Erickson, 1983, p. 723). The question is usually addressed in forms like "Is Jesus both finite and infinite?" or "Did Jesus, being God, know all things all the time?" The latter question is even more significant in light of the fact that Jesus Himself said that there was something He did not know. Mark 13:32 says, "But of that day and that hour knoweth no man, no, not the angels which are in heaven, neither the Son, but the Father" (KJV). Let us be more specific and address the issue of the deity of Christ as it relates to God's “omni” characteristics. Here are the propositions:

- Jesus is (and has always been) God.
- God is omniscient, i.e., knows all things all the time.
- Thus, Jesus is omniscient.

There is no question that the third sentence (conclusion) is a straightforward and valid statement deduced from the first two premises. It is a logical necessity. However, the conclusion clearly contradicts Mark 13:32, from which we get the negation of the conclusion: "There exists at least one thing which Jesus did not know, at (at least) one point in time." The logical problem is clear. Jesus is (at least was, at one point in time) not omniscient. If God is necessarily omniscient, this eliminates the possibility that Jesus has always been God.

Most evangelical Christians are not willing to give up the deity of Christ or the omniscience of God. Hence, in light of Mark 13:32, perhaps one must modify the definition of omniscience. But doing so suggests that God may not know all things all the time, essentially rejecting one of God's infinite characters.

Historically, many attempts have been made to resolve the issue. Some said that Jesus had a divine soul and human body (Apollinarianism). Another school held that Jesus the man became divine at a point during his lifetime (adoptionism). Others stated that Jesus had two natures before incarnation but just one after incarnation (Eutychianism). Kenoticism holds that Jesus' incarnation was "an exchange of part of the divine nature for human characteristics" (Erickson, 1983, p. 733). Dynamic incarnation teaches that the incarnation was "the active presence of the power of God within the personal Jesus" (Erickson, 1983, p. 733). The list goes on and on.

The post-Cantorian foundation of mathematics suggests the following: if the axioms suggest that God is omniscient and that He is not, then the axioms are inconsistent. Hence, the options appear to be rather bleak. Unless one is willing to give up the deity of Christ or the consistency of the Scripture, one must accept the logically necessary consequence: it is possible for a divine person not to possess all knowledge all the time.

Just as God's omnipotence does not imply that God can do anything at any time (for instance, He cannot lie or become a non-divine being), is it possible that His omniscience does not imply that He knows all things all the time?[31] Traditional theology may consider this a heresy. In light of the lack of other options, however, one needs to examine this statement to see if it really contradicts the biblical evidence. For instance, Psalm 147:5 says that "His understanding is immeasurable," but clearly knowledge can be immeasurable without being all-inclusive. Thus God may know the number of stars (Ps. 147:4), life of every sparrow (Matt. 10:29), number of hairs on every person (Matt. 10:30), etc., without knowing everything all the time. Therefore, it appears that such a position, while negating the traditional idea of omniscience, could provide a possible solution within the "biblical axiomatic system." By no means do I claim to have resolved the mystery of hypostatic union. I am, however, suggesting that the problem be re-examined with these possibilities.

Sometimes the question of hypostatic union is addressed in this way:

- Man has finite knowledge.
- God has infinite knowledge.
- Jesus is both God and man.
- Hence, Jesus has both finite and infinite knowledge.[32]

The last sentence is self-contradictory but a logically valid conclusion deduced from the first three propositions. Because the logic is flawless but the argument is not sound, the problem must be in at least one of the premises.

A few comments on this question seem appropriate here. First, is man's knowledge really finite? Often the answer is like the following response I heard recently: "Sure; otherwise, man would be God." But such a response clearly presupposes that man is not God, or more precisely, God and man are mutually exclusive. Hence, it does not apply if the particular man involved here also happens to be God. Or, is it possible that man, by some rule or definition, must have finite knowledge? The fact is that man's knowledge need not be finite unless the finiteness is one of the defining properties of man. This is very similar to the sinfulness of man. (Being man does not require sinfulness.)

Perhaps a specific mathematical example may be helpful. Consider the unit circle, i.e., the set of all points z on the complex plane such that $|z| = 1$. Now define multiplication on this set in the usual way. Under this binary operation, this set is an abelian group. But why? Well, all one needs to do to prove this fact is to demonstrate that this set satisfies each part of the definition of an abelian group (associativity, commutativity, identity, inverse, etc.). Note that whether a set is finite or infinite is irrelevant.[33] In a similar way, there may be X such that X is man and X has infinite knowledge. Such a being X must satisfy all the defining properties of man while possessing infinite knowledge.[34] This is not a contradiction.

This points to the importance of clearly established definitions. What makes an object X a man? What makes a being God? (This turns out to be very important in the discussion of the Trinity below.) Granted, certain things and objects are left undefined in any axiomatic system, but if terms like God and man are used as undefined terms (like lines and points in geometry), then the set of axioms should provide enough descriptive properties about them (such as "Two points determine a line," where the terms "points" and "line" could mean whatever they may be within the particular geometry).

One additional thought that may be helpful in addressing the difficulty of hypostatic union is the theory of dimensions. Often we assume that every question in the form "Does X have property P or not?" must have either an affirmative or a negative answer. But one must remember that sometimes questions of this nature are inappropriate. (Do you ever say, “The answer is ‘Yes and No.’”?) For instance, consider a circular cylinder. Is it round or not round? Well, from one angle, it is round; from another, it is rectangular. For another example, consider the unit interval. What is its (Lebesgue) measure? In 1 dimension, the answer is 1; in 2 dimensions, it is 0. How about the vertical line $x = 3$? Its projection onto the x-axis is just one point (a very finite set) while its projection onto the y-axis is infinite and unbounded. In a similar fashion, perhaps Jesus, equipped with deity and humanity, may have an additional dimension so that the projection onto one coordinate is finite while the projection onto the other is infinite. Hence, the question “Is Jesus finite or infinite?” may not be an appropriate yes-or-no question.

(4) Trinity

The concept of the Trinity is perhaps the single most debated notion in the Bible. Some groups, such as Jehovah's Witnesses, totally reject the idea, partly on the basis of logic. Throughout history, various issues related to this teaching have created many divisions and cults. What is its teaching? In a nutshell, it teaches the following:

- There is only one God.
- God exists in three distinct persons: the Father, the Son, and the Holy Spirit.
- Each is fully God; they are not three parts of one God.

Because of the difficulty in grasping this, we often hear people say, "There is no way to explain it or illustrate it. Every analogy fails." The analogies referred to here include one about the three forms of water (ice, water, and steam) and another about the egg (with its three parts).

It is not surprising at all that every analogy “fails.” An analogy is an analogy not because every element has a corresponding element in the truth, but because there is at least one point of similarity between the analogy and the truth being illustrated. The same people who claim that every analogy fails for the Trinity would not hesitate to use other analogies, such as a slave market to illustrate redemption (where the analogy also miserably fails). Further, one must note that Jesus Himself used analogies in a masterful way even though every one of these analogies "fails" (for example, Jesus calls Himself the "bread of life," but there are many things true about bread that do not apply to Him). Therefore, the idea that there cannot be any good illustrations of the Trinity should not be easily accepted.

It is worthwhile here to compare what I am proposing—Post-Cantorian theology—with Pre-Cantorian theology. Take, for instance, the letter that John Adams wrote to Thomas Jefferson on Sept. 14, 1813: Adams writes (Cousins, 1988, p. 239):

> *Had you and I been forty days with Moses on Mount Sinai, and admitted to behold the divine Shekinah, and there told that one was three and three one, we might not have had the courage to deny it, but we could not have believed it.*

Jefferson also made his belief clear in his Nov. 4, 1820, letter to his friend Jared Sparks (Cousins, 1988, p. 156):

> *The metaphysical insanities of Athanasius, of Loyola, and of Calvin [referring to the Trinity], are, to my understanding, mere relapses into polytheism.*

These men, along with many of the rationalistic thinkers of their time, concluded that the Trinity implies that 1 is equal to 3 and thus rejected the doctrine altogether.[35] A. H. Strong, in his *Systematic Theology,* quotes A. J. Gordon as saying, "In mathematics the whole is equal to the sum of its parts. But we know of the Spirit that every part is equal to the whole" (1907, p. 281). This is a statement of pre-Cantorian theology.

In contrast, Cassius Keyser, in his article "The Rational and the Superrational" (1952), uses an illustration of infinite sets to demonstrate that parts could be just as large as the whole. This is now a rather well-known analogy among Christian mathematicians and has been quoted in print by numerous scholars. His illustration partitions the (countable) set of positive integers into three disjoint sets A, B, and C: A consists of the multiples of 3, B is the set of positive integers 1 mod 3, and C is the set of positive integers 2 mod 3. The union of these is the entire set of positive integers, but each is in one-to-one correspondence with the set of positive integers. Keyser adds that, while this does not explain away the Trinity, it shows that the concept of the Trinity is "rigorously thinkable, perfectly possible and rational." In other words, there are mathematical entities that behave somewhat like the Trinity and, therefore, the idea is not as "absurd" or "ridiculous" as once thought. The notion of one-to-one correspondence between sets is exactly what Cantor used to develop his theory of transfinite numbers. This then is an excellent example of the use of post-Cantorian theology.[36] One cannot help wondering what Adams, Jefferson, and others would have thought had they been introduced to these mathematical ideas of Cantor.

Here is another mathematical analogy coming from the construction of numbers. As explained above, in set theory, one can construct all positive integers from the empty set. This set can easily be expanded to the set of integers. To construct the rational numbers next, the standard method is to consider the set of ordered pairs *(a, b)*, where *a* and *b* are integers and *b* is non-zero. Then, one defines an equivalence relation on this set as follows: *(a, b)* = *(c, d)* if and only if *ad* = *bc*. The set of equivalence classes thus constructed is the set of rational numbers, where one interprets *(a, b)* as the fraction *a/b*. Here, *distinct* ordered pairs, *(1, 2)*, *(3, 6)*, and *(-4, -8)*, are actually the same rational number—sharing the same essential nature as a number. However, they are *distinct* ordered pairs and have different functions and roles. For instance, if this number is to be added to 5/2, we use the first representation (as they share the common denominator 2). Again, not every aspect of this analogy carries over to the spiritual sphere; however, note that it is possible for three distinct elements to share exactly the same essence.

I should mention here that this illustration, Keyser's "mod-3" example, and the example of a circular cylinder under the topic "Hypostatic Union" are all special cases of a larger, more general idea, called *equivalence relation*. It is my opinion that the concept of equivalence relation has a lot to say about various issues of theology. This needs to be further explored.[37]

Before concluding the section on this important doctrine, I should again stress the importance of definitions. When debating or discussing the Trinity, many people take definitions of key terms for granted. For example, one of the key words in the discussion of the Trinity is the word "person." What makes X a person? (For instance, when X is the Holy Spirit.) At this point we need a precise working definition of what a person is—precise in the sense that it includes all beings called "persons," including the Holy Spirit and God the Father but excludes all non-personal beings such as dogs.[38] Erickson refers to this problem without giving any answers (1983, p. 338). Berkhof, in his *Systematic Theology*, defines "person" as a "separate rational and moral individual, possessed of self-consciousness, and conscious of his identity amid all changes" (1938, p. 87). With this definition, he goes on to explain the doctrine in a manner easy for the reader to follow. This illustrates the fact that understanding the importance of definitions is a key in clearer, more precise understanding of various doctrines. As obvious as this fact is, theological and philosophical discussions often leave key terms undefined.

4. Notes on Mathematical Methods and Logic in Theology

As stated earlier, the primary purpose of this paper is to propose a way in which one approaches the discipline of systematic theology in light of the historical development of mathematics. An underlying assumption, which must be obvious by now, is that mathematics can and should influence theology. It is interesting to point out that there seems to be ongoing interplay between these two disciplines. Gene B. Chase wrote a paper exploring how theology has influenced mathematics (as opposed to how mathematics influences theology), studying the works of mathematicians like William of Ockham, Hamilton, Boole, and Cantor (1991, p. 191). He dismisses the question of the influence of mathematics on theology as "less interesting," citing works already done by Granville Henry, who, for instance, suggests that non-Euclidean geometry gave rise to the theory of relativity, which Whitehead used to develop his process theology. Regardless of which direction of this "influence" or "interplay" is more *interesting*, the existence of such a mutual relationship between mathematics and theology seems undeniable.

Many mathematicians who share faith in Christ and interest in theology have come up with insightful analogies and arguments to better understand theology. For example, Thomas Iverson, in the very first conference of the ACMS (Association of Christians in Mathematical Sciences), presented the paper "God: All Sufficient or Infinite." There, he compares hypostatic union with the stereographic projection and suggests that the issue of free will vs. sovereignty may be undecidable (1977, p. 124). Robert Brabenec uses the idea of infinite limit to illustrate perfection, discusses Cantor's paradox and the term "all" in the Bible, and suggests an isomorphism between heavenly and earthly realms (1983, pp. 87—88). One can probably add many other examples to the list of mathematical concepts that can illustrate or model biblical and theological teachings.

These analogies or illustrations are very helpful and beneficial, at least in my opinion. However, the central theme of the present paper is more general: the method I am proposing here is an axiomatic, mathematical approach to theology. It uses not only the contents of mathematics as analogies but also the structure of mathematics. Regarding the use of logic and mathematics in theology, various people have expressed their opinions and concerns. For example, Jan de Koning, in his 1991 paper, warns that "mathematical methods are often used in wrong places" and that "it is wrong, even dangerous, to use mathematical methods for studying theology" (1991, pp. 176—177). The primary example he uses is the argument between Arminius and Voetius, which ended up splitting the Church. A similar warning was stated centuries ago by Thomas Aquinas, who held the following view with a pre-Cantorian flavor: "Mathematics must concern itself only with the potentially infinite. For God represents the only actual infinity, and God is not an object of mathematical study."

Although these warnings are, to a certain extent, worth considering, I maintain that mathematics and logic remain to be effective tools in studying theology, as long as it is properly applied. In the case between Arminius and Voetius, cited by De Koning, the problem was the points each of these men wanted to stress, partly influenced by their own background. The problem that split the Church was not caused by mathematics or the use of it in theology. As to the statement of Aquinas, unlike his days, today (thanks to Cantor) we have a rather sophisticated theory dealing with actual infinity—or actual infinit*ies*. These concepts are too good to be excluded from the study of what Aquinas called the "only actual infinity," God.

De Koning further says, "Too often the problem is that Christians try to put God in a box." He then adds the following remark to explain what he believes is a dangerous idea (1991, pp. 184—185):

> *This community [of believers] would also remember that mathematical logic will not work for theology, simply because mathematics is time- and creation-bound, and God is not.*

This argument is inaccurate for a number of reasons. The most obvious one is that the reason given here (the bounded nature of mathematics) fails to establish the conclusion (invalidity of logic in theology). Such an argument would also negate the validity of anything else (e.g., human language) in theology. Note that the causal relation referred to in this sentence is itself a logical structure. In fact, not everyone holds that mathematics is bounded in this sense. Some argue that the fact that God exists in three persons from eternity past implies that the abstract concept "3" (neither the set-theoretical construction of the number, nor the numeral that denotes this concept) has existed from eternity past.

In addition, classical logic, including the law of excluded middle, is implicit throughout the Scripture. This is essentially the central point of a chapter in John Byl's book *The Divine Challenge* (Ch. 14). There, Byl states clearly: "Logic is not above God, but derives from God's constant and non-contradictory nature" (2004, p. 257). If one does not accept the validity of logic in the theological realm, there cannot be any systematic theology, and the Bible itself becomes untrustworthy, for no one can deduce *any* conclusions based on a biblical teaching. For instance, if the law of excluded middle is not true, while God is love, it is also possible that God is *not* love. If mathematical logic is part of God's nature as Byl holds, then it cannot contradict any truths of God.

Finally, mathematics and logic are used universally to model all kinds of phenomena in virtually every discipline. When theology is thought of as a science, it should not be surprising that mathematics and logic can be used to find truths in this particular area, just as in any other academic discipline. Hence, while I agree that mathematics and logic must be used with great care when applied to theology, I assert that they are legitimate, valid, and effective tools in exploring theological issues.

5. Conclusion

Some contemporary theologians are quite competent and knowledgeable in mathematics, skillfully applying mathematical notions in theology. For instance, William Craig argues against fatalism using Newcomb's Paradox, which uses the theory of probability and expected value (1987). Richard Swinburne, an Oxford philosopher, uses formal logic, Bayes' Formula for conditional probabilities, and axiomatic probability theory to establish the historicity of biblical events, including resurrection (2003). Walter Kaiser uses what is called *syntactical display* (logical display similar to how formalist mathematicians tried to encode all of mathematics) to analyze the entire book of Malachi (1984). There are others as well, and my hope is that more theologians will learn more mathematics and more mathematicians will learn more theology so that the beautiful connection between these two disciplines can be realized and pursued to the maximum potential and will produce not only a high level of scholarship but also truly edifying results and consequences.

6. Acknowledgments

I would like to express my sincere appreciation to the referees of this article for invaluable suggestions and advice. I also thank the Board members, leaders, and fellow members

of the Association of Christians in Mathematical Sciences for their encouragement, support, and prayers.

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Endnotes

[1] One of the major problems in the philosophy of mathematics is to produce a good definition of mathematics. Many attempts have been made, and mathematicians are still presenting possible solutions (e.g., Bonnie Gold of Monmouth University, "*What Is Mathematics II: A Possible Answer,*" at the MAA Meeting in Atlanta, Jan., 2005).
[2] S. Matsumoto, *Strong Integrability of Mathematics with the Christian Faith*, in preparation.
[3] This is *not* to negate the role played by the Holy Spirit in illuminating each believer as he or she reads the Bible. However, my thesis is that mathematical and logical methods could contribute significantly in our understanding of the Scripture. See Section 4: "Notes on Mathematical Methods in Theology."
[4] For instance, both Yale and Harvard began using Euclid's *Elements* in the 1730s (Burton, 2003, p. 535).
[5] For this remarkable treatise, Euclid is called the "Great Geometer" throughout history.
[6] I could not recall where I read this quotation, but a referee has pointed out (and I have since verified) that this quote can be found in the online version of *Encyclopedia Britannica*.
[7] To put these men in a historical context, all of them lived long before Descartes introduced his coordinate system.
[8] Incidentally, Lambert is the first one to prove that π is irrational (1768).
[9] A possible exception may be Saccheri, who declared that his work was "sufficient to clear Euclid of the faults with which he has been reproached" (Burton, 2003, p. 529). Ironically, it was his failure that maintained Euclid's reputation, as we shall see later.
[10] The quote is from H. Meschkowski's book *Noneuclidean Geometry* (1964), quoted by Greenberg, 1993, pp. 161—162.
[11] It is very interesting, from a philosophical standpoint, that the young Bolyai used the two verbs that represent opposite views of the existence of mathematical objects: "discover" and "create." Perhaps he meant that, while he did *create* a model of hyperbolic geometry, the possibility of a consistent geometric system without the Parallel Postulate was *discovered*.
[12] Although Bolyai was the first to mention non-Euclidean geometry in a private letter (1823), Gauss appears to be the first to have considered it (1817), and Lobachevsky was the first to publish it (1829).
[13] Indeed, Gauss considered Wolfgang Bolyai so close that Gauss wrote a personal letter to Bolyai about his happy engagement at age 28 (Bell, 1937, p. 243).
[14] The full body of the appendix was 24 pages long. These are the only pages John Bolyai ever published, but they are called the "most extraordinary two dozen pages in the whole history of thought" (Burton, 2003, p. 549). Incidentally, the book with this appendix was sent to Gauss twice as the first one got lost in the mail, never reaching Gauss.

[15] In fact, when his father's book *Tentamen* was translated to Hungarian from the original Latin in 1834 (only three years after the original publication), John's *Appendix* was omitted altogether. It was not recovered until 1867, seven years after John Bolyai's death (See Burton, 2003, p. 548).
[16] It has been said that, if someone had asked I. Kant if he could imagine non-Euclidean geometry, his reply would have been, "I Kan't."
[17] The Hyperbolic Parallel Postulate states that there is a line L and a point P not on L such that there are two lines parallel to L through P.
[18] Actually, for that matter, Euclidean geometry was not proved consistent, either.
[19] There is a humorous way to describe the two geometries involved here: A Euclidean baseball team and a hyperbolic baseball team were to play a game against each other. Upon coming to the field, the Euclidean team was shocked to see that the home plate is a pentagon with five 90-degree angles. The hyperbolic baseball team was shocked to see that the "diamond" was a complete square. Neither team could hit very well because they kept thinking that straight balls were curves and curves were straight.
[20] Mysteriously, Cantor, a Russian, wrote this sentence to his German friend Dedekind in French. At the 2005 conference of the ACMS (Association of Christians in Mathematical Sciences) held at Huntington College in Huntington, IN (where this paper was first presented), it was pointed out by Francisco Gouvea, a plenary speaker, that perhaps this sentence is often used out of context and that Cantor may not have been as surprised as we have always been told.
[21] Russell's paradox turned out to be very bad news for Frege; he was about to publish a book whose entire contents would have been wrong because of Russell's paradox. Byl (Ch. 8) quotes Titus 1:12—13 as a New Testament example of this paradox, which eventually put an end to the noble but doomed efforts by the once-optimistic formalist school of mathematics, led by Hilbert.
[22] Carol Schumacher, in her textbook (2001, p. 197), quotes a theoretical physicist who said that "all of mathematics is composed entirely of brackets—take the brackets away, and there's nothing left."
[23] We know today that this theorem is logically equivalent to AC.
[24] Paul Cohen, a Fields medalist, now retired, was one of the panelists in a seminar held at the Joint Meetings of the American Mathematical Association and the Mathematical Association of America, held in Atlanta, in January, 2005. His opening remark made it clear that he is not a realist, at least in the ontological sense: "First of all, let me just say that I don't believe that sets really exist."
[25] Paul Gordan was the prominent algebraist under whom Emmy Noether studied. Gordan is sometimes referred to as the "King of Invariants" (Burton, 2003, p. 683). On Hilbert's approach, Gordan later admitted that "theology has its merits" (Burton, 2003, p. 581).
[26] Hilbert, *The Foundations of Mathematics*, quoted in Byl, p. 133.
[27] For a detailed explanation of the various solutions to this problem, see, for instance, Erickson, Ch. 19.
[28] Obviously, the *real* death of Mary Baker Eddy somewhat weakened this position.
[29] I have been advised that this may be a biased statement, i.e., such an algebraic approach may appeal to me only because I am a mathematician. However, one of the goals of this paper is to show that this approach helps anyone, mathematician or not, understand theological arguments.
[30] The term "neutral geometry" refers to the geometry with all the usual axioms except the Parallel Postulate.
[31] This resembles the main problem in Cantor's Paradox: the use of the term "all" in naïve set theory.
[32] One must be careful to distinguish between "infinite knowledge" and "omniscience." The former simply means "knowledge that is not finite" while the latter means "knowing everything." One cannot be omniscient if he lacks even one piece of knowledge, but he can still possess infinite knowledge (provided that there are infinitely many things to know). A set can be infinite without being the universal set if the universal set is infinite.
[33] Incidentally, the unit circle is infinite. In fact, it is uncountably infinite as a set although it is clearly bounded on the plane. Being bounded or not is another factor that is irrelevant to the definition of a group, which is purely algebraic.
[34] If God has infinite knowledge, having only finite knowledge is a sufficient (but not a necessary) condition for X *not* to be God. To say it is a necessary condition would be a fallacy of denying the antecedent.
[35] The Trinity does not teach 1 = 3; that would demolish all of arithmetic and mathematics.
[36] Keyser also used ideas like dimensionality, hyperspaces, and non-Euclidean geometries to illustrate theological concepts.
[37] When, for instance, one says, "Your book is the same as mine," the term "same" refers to the understood equivalence relation. The two books involved are clearly distinct objects (thus the adjective "two"). Equivalence relations provide a perfect example of considering (identifying) two or more things (in the case of the Trinity, three)

as one. I am not prepared to claim that the Trinity is an equivalence class (at least not yet), but this needs to be examined further.

[38] Such a definition would help decide whether or not angles are persons.