

Mathematics at Chartres Cathedral

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Having had several opportunities to travel to France, often with groups of students, our trips have usually included a visit to Chartres, especially to visit the magnificent Gothic cathedral that dominates the town. On a recent visit I was again struck by the beauty, majesty and awe that the cathedral inspires. The building not only does a remarkable job of telling Biblical stories and of enclosing a space conducive to worship, it directs one's eyes and one's spirits upward. This is achieved not only by the beautiful stained glass windows and the striking sculptures, but also by the overall design and proportions of the structure. When I returned home I thought it would be interesting to read more about the cathedral. It was then that I discovered that, in at least some authors' minds, mathematics, particularly geometry, had a central role to play in the design and construction of the building. It is this contribution of mathematics that I would like to share in this paper.

The town of Chartres is located approximately 50 miles southwest of Paris. There has been a church in Chartres since the eighth century, however, it was in 876 that the town and its church became an important site. In that year Charles the Bald, grandson of Charlemagne, made a gift to the church at Chartres of the Sancta Camisia, a relic that was believed to be the veil or tunic worn by Mary when she gave birth to Christ. From that time on the church not only served to meet the religious needs of the townspeople, it was also used to house this relic and to accommodate great numbers of pilgrims who would come to Chartres to pay homage to the veil. Because of this relic, the church took on a special connection to the Virgin Mary.

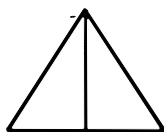
In the Middle Ages, the town of Chartres was also an important seat of learning. In 990 a scholar named Fulbert came to Chartres and started a school which would rank as one of Europe's leading scholastic institutions for the next 200 years. As Malcolm Miller points out in his booklet, "Chartres Cathedral", the liberal arts were prominent in the school. Quoting the scholar Thierry of Chartres Miller notes, "Philosophy has two principal instruments, the mind and its expression. The mind is enlightened by the Quadrivium (arithmetic, geometry, astronomy, and music), its expression, elegant, reasonably ornate, is provided by the Trivium (grammar, rhetoric and dialectic)." Today there is a visual reminder of the school and the importance of learning amid the sculptures of the cathedral. The Royal Portal, located at the base of the western wall of the cathedral, is the entrance that most people use. Of the three bays that comprise this entrance, the right bay is dedicated to Mary. Above the door, the Tympanum tells the story of Mary. These sculptures are in turn surrounded by symbols for each of the seven liberal arts, along with a sculpture of a famous practitioner for each. Interestingly, Pythagoras is included as one of these practitioners.

The present cathedral was built over a thirty year period, begun in 1194, almost immediately after a devastating fire destroyed most of the previous building. It is generally agreed that the guidelines for constructing the new cathedral were determined

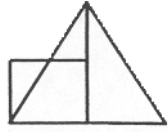
by the Chapter, as they were responsible for providing for the many uses of the building. However, once the Chapter determined the basic guidelines, it was up to the master builder to execute the plans and assume responsibility for the design and construction of the building. To the regret of many historians of art and architecture, the identity of these master builders is unknown. However, there is strong agreement that these men had remarkable skill as designers, as problem solvers, and also as practical engineers. They were not merely technicians who were adept at cutting stone, mixing mortar, and bringing different facets of building processes together. These were the people who functioned as the architects, responsible for maintaining a vision of the finished project, even while being concerned with the myriad of details involved in the construction process.

As might be expected, constructing a cathedral in the thirteenth century was a much different process than what we might expect. For one thing, the work was not always one continuous process; construction was often halted for one reason or another. For example, there were likely to have been periods when they exhausted the funds to pay for workers or supplies. At those times work would have to cease until funds became available. Breaks in construction could mean that the master builder had to move to another project, then be replaced when construction resumed. For that and probably other reasons, there was almost never one single master builder who was responsible for a project from beginning to end. Rather, new masters would come on the scene and they would have to adapt their plans to what was already constructed. In [4] James claims to have identified nine different master builders who worked at Chartres. Of course, these nine did not all exert the same influence on the final product. However, the fact that there was this change of leadership did affect the final product. Interestingly, these masters did not all use the same standard of measurement. Thus it was likely that new masters would have to deal with the added complication of continually making adjustments in their calculations, based on what was done before. Nevertheless, despite the fact that the process was done over a thirty year period (actually, this was a remarkably short span of time for building a cathedral) and that the project was under the direction of different people, the final result stands as a magnificently constructed building that is unified through a variety of themes and plans.

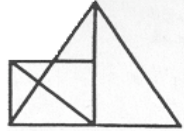
One way that we would expect mathematics to play a role in the cathedral is by the nature of its use generally in engineering and construction. Calculating dimensions, constructing parts such as flying buttresses, and even designing the tracing in the windows all made use of mathematics. I was particularly impressed by one example where James explains an elaborate plan for the construction of a window onto a circular stairway. An important aspect of this makes use of the ratio $\sqrt{2} : \sqrt{3}$. Of course this ratio, which at first might appear somewhat formidable in terms of measurement, can be fairly easily constructed using an equilateral triangle and a square. Given an equilateral triangle, construct an altitude of the triangle



and a square on half of one side.



Then the ratio $\sqrt{2} : \sqrt{3}$ is simply the ratio of the length of the diagonal of the square to the altitude of the triangle.



Examples like this illustrate that the master builders were also quite well-versed in geometry. However, apart from the expected uses of mathematics to solve typical engineering problems, there is an interesting use of mathematics and mathematical relationships in the design of the cathedral. We will first consider the motivation for this, then look at some examples of how this is done.

The cathedral was not designed or built in a cultural vacuum. It is interesting to note that both Panofsky [7] and Raddings and Clark [8] make connections between learning and architecture. Although these authors do not agree at every point, they do each establish this link. For example, each points out that the scholastics were the first to organize arguments into sections and sub-sections. This is seen in contrast to the “books” that were previously used (one is reminded even of Euclid). This tendency is manifested architecturally by the uniform divisions and subdivisions of the space. Clearly the floor plan of Chartres embodies these subdivisions. Using similar arguments, these authors argue for a strong connection between the state of learning and architectural style during this period.

Recall that by the time work on the present cathedral was begun (1194) the school at Chartres had a well-established tradition. Otto von Simson [9] points out that at the time of the building of a new cathedral there was already a strong Neoplatonic tradition at the Chartres school, a tradition that focused around mathematical and musical studies (p. 188). He notes, (p. 27) that “The masters of Chartres, like the Platonists and Pythagoreans of all ages, were obsessed with mathematics: it was considered the link between God and the world, the magical tool that would unlock the secrets of both.” He goes on to give examples of how the scholar Thierry of Chartres used geometry and arithmetic to help bring out aspects of theology. For example, von Simson points out that for Thierry, the equality of the three persons of the Trinity is represented by the equilateral triangle. Thus, at Chartres, anyway, there is already a strong disposition toward a high view of mathematics and we should not be surprised to see this incorporated into the building. However, this was not peculiar to Chartres. In many Gothic Cathedrals the intentional use of mathematics, and geometry in particular is evident in the design process. As Harvey points out (p. 85), “. . . geometry was considered fundamental both in religion and in mundane affairs, even though practical

knowledge of its operations was limited.” He later states “the activities of the Creator were reflected and echoed in the works of human artists, and those works depended for their utility, durability, and beauty upon properly composed geometrical relations.” This high view of geometry is connected to the view of God as the creator of the universe and the creator of man. Harvey further points out (p. 37) that geometry was seen as the “fundamental harmonizer of the universe and of human life.”

We will soon consider several examples, closely following those given by James, of how geometry was used in the design of the building. Before doing that, consider one other piece of evidence of why the principles of geometry were deliberately incorporated into the design of the building. This quote is from James [4], p.66.

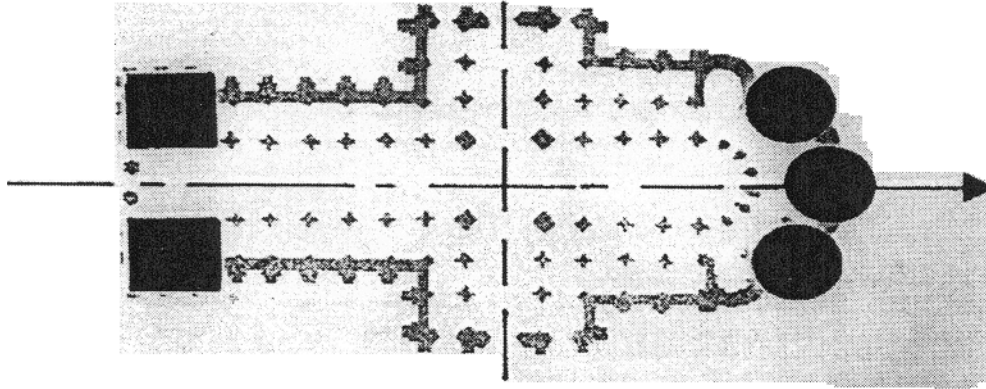
“In the middle ages people did not see that function was the only attribute required of a building. Beyond the events and needs of our world lay the hierarchies of other beings and existences. This is not something we readily understand today, for where science provides the answers there are no mysteries. They believed that behind the superficial appearance of things lay a greater reality that would, if only it could be tapped, reveal the true nature of the universe. The ultimate mystery was as real to them as the laws of nature are to us. They not only tried to find ways to understand them, but, as we do with scientific discoveries, attempted to apply what they had learnt to what they did. The results of their investigations were expressed in numbers and geometry, not unlike our formulae. They were applied to building so that the structure would reflect the Divine, and by reflecting it, illuminate man’s way.”

James amplifies this (p. 85) when he says, “The actual world that forms the basis of our reality was for them merely a reflection, and often a poor one at that, of the more important spiritual world.”

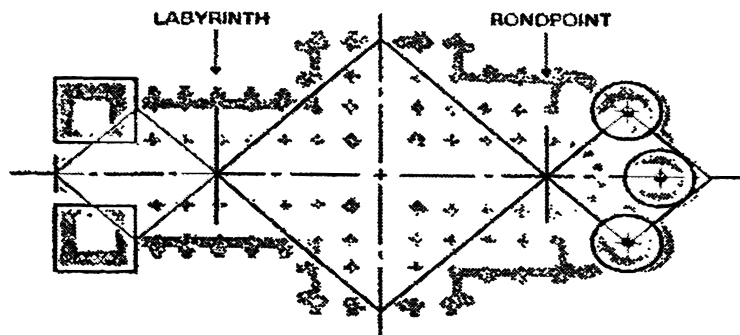
Given the large place that number and symbolism in general played in the lives of the people of the Middle Ages it is inconceivable that a cathedral would be built without incorporating such symbolism. As James says (p. 85),

"In our day we call the church the House of God, for his presence occupies it. But the thirteenth century was less circumspect. They had the audacity to believe they were constructing a slice of eternity itself, and the simplicity to trust that God’s Essence would be made manifest in something they built from the materials found on the earth. They achieved this by setting into the design as many of the appropriate numbers and symbols as possible, so that the different levels of meaning tumble over one another, layer upon layer, until the mind reels under the weight.”

As an example of how mathematics, and particularly geometry, was used in the design of Chartres, consider the design of the floor plan. As with most cathedrals, the floor plan at Chartres is in a cruciform shape, with the nave and choir lying along the east-west axis and the transepts along the north-south axis.



There are two towers at the western end and seven chapels, three larger and four smaller chapels, around the ambulatory at the east end. As James describes the floor plan, the original master builder made use of three diamonds, as seen below, which give a special symmetry to the design.



Note that the vertices of these diamonds identify the western entrance, the entrances in the north and south transepts, the centers of two of the larger chapels and the eastern endpoint of the cathedral. The diamonds meet at the center of the labyrinth, a huge pattern inscribed in the nave which might symbolize man's way to God, and the rondpoint, the curved eastern end which separates the ambulatory (the curved path around the apse) from the main portion of the church. I appreciate this figure for its simplicity and its completeness.

As a second simple example of how geometry was used in the design of the cathedral, James, along with other writers, indicates that regular figures such as pentagons and hexagons figured prominently in the design phase. Because they have six sides, and six is a perfect number, hexagons were intriguing to medieval mathematicians and were included in the planning process. James contends (p. 11) that the original master builder even used a hexagon to determine the position of the transept walls.

James goes on to describe in greater detail many other geometrical relationships contained in the design of the floor plan. He also details how geometry was used to construct the elaborate tracery in the windows, and to detail complex number relationships seen in the cathedral.

These examples are given primarily to lend credence to the fact that geometry was an important factor in the construction of this cathedral. We have come a long way since those days in the design of our churches. It has been interesting to me to realize the central role that mathematics, and particularly geometry, played in the design of these special buildings. Perhaps the fact that mathematics was so intentionally incorporated is one of the important reasons why these cathedrals remain important to us so many years later, not only as objects of art, but also as important places of worship.

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