Creationism – A Viable Philosophy of Mathematics

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1. Introduction

The purpose of this essay is to try to answer the ontological and epistemological questions of mathematics. Specifically, "What, if any, of mathematics exists in the objective sense?" And, "How do we as humans know that our knowledge of mathematics is correct?" These questions will be investigated by looking at the applications of mathematics, the practice of mathematicians, and most telling, the content of mathematics. Mathematics, admittedly, can only go so far in answering its own philosophical questions, even when aided by recent developments in the field of logic. The overwhelming evidence, as will be shown, points toward a theistic, or more precisely, a creationist, interpretation of mathematics. The presuppositions of Christianity will be shown to have the powerful ability to fill in the philosophical gaps left by mathematics, legitimately addressing the existence and knowledge questions.

2. Creationism defended

Premise 1: Mathematics is real, at least as a real description of the universe. Mathematics has been used to model and describe countless phenomenon observed in business and scientific applications. For example, consider models of two variables x and y.

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$
$$y = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$$
$$y = a + b\ln x$$
$$y = ax^b$$
$$y = ab^x$$
$$y = \frac{c}{1 + ae^{-bx}}$$
$$y = a + b\sin(c(x - d))$$

These common "mathematical models" are referred to as second-degree, polynomial, logarithmic, power, exponential, logistic, and sinusoidal. These seven models have been used to describe and predict the accumulation of wealth, cost analysis, marketing strategies, radioactive decay, the earth's mass distribution, the orbital paths of celestial bodies, and population fluctuations, just to name a few. What is so remarkable about this list is the simplicity of each equation. The only requirement is knowing the value of a small number of constants. The universe seems "hard-wired" with the laws of mathematics. The applications of mathematics draw one to conclude that mathematics must be real at least as a description of the physical universe.

Premise 2: Mathematics exists independently from the physical/temporal universe. If one talks with mathematicians at any length one is left with the impression that mathematicians think that mathematics is of utmost importance. It is not just that mathematics is simply describing something. They treat mathematics as if it were a real entity, whose truth does not depend on the physical universe. Menzel (Howell & Bradley, 2001, p. 69) notes that even if there were not "pebbles and pomegranates and other countable sorts of things, there still would have been the number 11, as well as the proposition that it is prime." Also implied is the independence of mathematics from time itself. The theorems of mathematics seem as if they were true even before time began and would continue to be true though time no longer existed. The practice of mathematics draws one to conclude that mathematics is true and exists independently from the physical/temporal universe.

Premise 3: Mathematics is communicable between rational beings. First-order logic has been used as a foundation for almost all of mathematics. In particular, first-order logic includes the formalities needed to insure the effectiveness of formulas, axioms, and rules of inference.

i) For a formula to be effective "there must be an effective procedure for deciding, for an arbitrary string of symbols, whether it is a formula."

ii) For an axiom to be effective "there must be an effective procedure for deciding, for an arbitrary formula, whether it is an axiom."

iii) For a finite sequence of formula, "there must be an effective procedure for deciding …

whether each member of the sequence may be inferred from one or more of those

preceding it by a rule of inference" (Stoll, 1963, p. 373).

First-order logic is built up with a number of allowable symbols:

i) variable symbols (like a_1, a_2, a_3, …)

ii) logical connectives: □, , □, □ , □ , etc.

iii) quantifiers: □, □

iv) equality: =

v) parentheses: (,)

vi) a set of constant symbols

vii) a set of relation symbols

viii) a set of functions symbols

0-ary relation symbols are allowed and are called propositional symbols. We specify a language in first-order logic by listing all other nonlogical symbols. For example arithmetic in the natural numbers can be specified using the language {0, s, +, *}, where s is the successor function: s(x) = x + 1.

First-order logic also specifies the rules that guide the construction of proofs. The rules are called the set of proof axioms for a first-order language, where x and y are variables and p,q and r are formulas:

i) $p \rightarrow (q \rightarrow p)$

ii) $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$

iii) $\neg\neg p \rightarrow p$

iv) $(\forall x)(p \rightarrow q) \rightarrow (p \rightarrow (\forall x) q)$, where x is not free in q

v) $(\forall x)(p \rightarrow p[t/x])$, where x is free in p, and t is any term whose free variables are not bound in p

vi) $(\forall x)(x = x)$

vii) $(\forall x \forall y)((x = y) \rightarrow (p \rightarrow p[y/x]))$, where x is free in p, and y is not bound in p

We are also allowed two rules of inference, commonly referred to as *modus ponens* and generalization:

i) from p and $(p \rightarrow q)$ infer q, where q has a free variable or p is a sentence

ii) from p infer $(\forall x)(p)$, where x does not occur free in any premise which has been used in the proof of p

These proof axioms require the notions of *free* and *bound* variables along with variable substitution.

> Informally, any occurrence of a variable in a formula in which $\forall$ does not appear is free; and in $(\forall x)p$ the quantifier 'binds' all occurrences of x in p which were not previously bound… Formally, if p is a formula, t a term and x a variable, we define p[t/x] ('p with t for x') to be the formula obtained from p on replacing each free occurrence of x by t, *provided* no free variable of t occurs bound in p; if it does, we must first replace each bound occurrence of such a variable by a bound occurrence of some new variable which

didn't occur previously in either p or t. (Johnstone, 1987, pp. 19-20, 22-23)

Gödel and Henkin were able to show that even in the broad context of all first-order theories, there were conclusions that could be made. The following meta-theorems (theorems about mathematics itself) were established, where T is a set of sentences for some first-order language L and p is any formula.

i) (Soundness Theorem) If p can be proven to be true, then p is true for every model of T with every assignment of variables.

ii) (Completeness Theorem) If p is true for every model of T with every assignment of variables, then p can be proven to be true.

iii) (Extended Completeness Theorem) T is a consistent theory (never leading to contradictory theorems) if and only if T has a model.

iv) (Compactness Theorem) T has a model if and only if every finite subset of T has a model.

Barwise (1977, p. 41) describes the accomplishments of first-order logic:

> Many logicians would contend that there is no logic beyond first-order logic, in the sense that when one is forced to make all one's mathematical (extra-logical) assumptions explicit, these axioms can always be expressed in first-order logic, and that the informal notion of *provable* used in mathematics is made precise by the formal notion *provable in first-order logic*.

Barwise claims that expressibility is supported by empirical evidence, and provability is supported by the completeness theorem. This means that rational beings have a mechanism for expressing mathematical ideas and for checking each other's proofs. Recent advances in logic draw one to conclude that mathematics is communicable between rational beings.

<u>Premise 4: Mathematics must be mediated by a time-independent intelligent being</u>. Hilbert tried to go a step further. He attempted to demonstrate that logic could also verify that there were no contradictions in mathematics. Martin (1977, p. 825) translates Hilbert as saying:

> If the arbitrarily given axioms do not contradict each other through their consequences, then they are true, then the objects defined through the axioms exist. That, for me is the criterion of truth and existence.

Hilbert attempted to define truth based solely on axioms, and hence to separate the truth of mathematics from rational beings, including God Himself. In others words, Hilbert is claiming that mathematics is created/sustained simply because it is non-contradictory, with no need for an intelligent creator. However, such a program could not be carried out, even on the natural numbers. Peano attempted to characterize the natural numbers with the following first-order theory, known as Peano arithmetic.

i) $\forall x \neg (s(x) = 0)$ (0 is not a successor)

ii) $\forall x \forall y\, (s(x) = s(y) \rightarrow x = y)$ (unique successors)

iii) $\forall x\, (x + 0 = x)$ (0 is additive identity)

iv) $\forall x\, ((x + s(y)) = s(x + y)$ (distributivity of addition successor function)

v) $\forall x\, (x * 0 = 0)$ (multiplicative property of 0)

vi) $\forall x \forall y\, (x * s(y)) = (x * y) + x$ (distributivity of addition over successor function)

vii) If f(x) is a formula with x free, then $f(0) \wedge \forall x\, (f(x) \rightarrow f(s(x))) \rightarrow \forall x\, f(x)$
(axioms of mathematical induction)

Gödel and others were able to show the fallacy of Hilbert's view of mathematics even when studying something as simple (?) as the natural numbers. The following theorems concern Peano arithmetic and can be used as evidence to support the necessity of a time-independent rational being.

i) (First Incompleteness Theorem) Any formal theory which contains Peano arithmetic cannot prove some sentences which are known to be true. This seems to indicate that mathematical truth is ultimately not simply a logic consequence of a set of axioms, but must be decided by some external intelligence.

ii) (Second Incompleteness Theorem) Peano arithmetic cannot prove a sentence that asserts the consistency (free from contradictions) of it own theory. This seems to indicate that the consistency of mathematics must be accepted on faith.

iii) The set of true sentences of Peano arithmetic is not definable by a first-order formula. This seems to indicate that a time-dependent being could never know everything there is to know about mathematics and therefore mathematics must have been created by a being outside of time.

iv) There is no effective procedure to determine whether an arbitrarily given statement is true or false. This seems to indicate that the verification of mathematics cannot be accomplished using only concrete methods. Since mathematics needs abstract methods, it also needs an intelligent being to use those methods.

v) Using a Turing machine, there is a sentence having n symbols in the theory of the multiplication of natural numbers that would take at least $2^{2^{2^{cn}}}$ steps to determine whether it was true. A Turing machine is an imaginary device that formalizes the mathematical concept of recursiveness. Taking c =1, then there is a sentence with only ten symbols concerning the multiplication of natural numbers that will take at least $2^{2^{2^{10}}}$ steps to determine whether it is true. This demonstrates the complexity of even simple proofs using a Turing machine. Church's thesis, which is almost universally accepted, claims that recursiveness is the correct formalization of computability. This demonstrates the complexity even of arithmetic itself. Post (1936) takes Church's thesis as describing humanity's mathematical power when verifying theorems. This demonstrates the limitation of our mathematical power even when it comes to arithmetic itself. This seems to indicate that a being having infinite knowledge is needed to fully comprehend mathematics.

vi) Any first-order theory of the natural numbers will have models of any large cardinality. This indicates that first-order theories produce multiple models and cannot "create" specific models, the models must have been created by an intelligent being.

The evidence is overwhelming. Mathematics exists beyond logic and even beyond human thought. Mathematics must be mediated by an intelligent being (or beings) who exists beyond time and space.

Premise 5: The God of the Bible is the creator/sustainer of mathematics. We have concluded that mathematics is objectively real and that its existence demands a time-independent being. Without invoking any particular religious tradition, we are essentially at a theistic perspective on mathematics. However, understanding the nature of God based upon the implications of mathematics is a much harder task. We might be able to conclude that God is logical because He is the creator of mathematics, and all of mathematics follows the rules of logic. This will not be the approach here. We will require the authenticity and authority of the Bible to go any further in this discussion (for example, see Moreland, 1987).

The Biblical record is clear to demonstrate that God values numbers. His counting brings meaning and significance to His created order. God's declaration of "And there was evening, and there was morning – the first day… And there was evening, and there was morning – the second day"

(Genesis 1: 5,8) sets the progression of creation in motion. For Christians, God's counting is what gives the believer a place in heaven. Jesus compared himself to a shepherd who leaves the ninety-nine sheep to rescue one sheep that is lost. He values each and every one of us.

The existence of the natural numbers is strongly supported Biblically. The Bible contains numerous references to mathematical objects, even what seem to be mundane measurements and counting. Numbers are used for describing dimension, notably for places of worship. God repeated specific dimensions from the building of a place of worship: God gave Moses dimensions for the tabernacle (Exodus 25-27), God gave Solomon dimensions for the temple in Israel (I Kings 6), and God has revealed the dimensions for the temple that is to be built after the return of Christ (Ezekiel 40-43). Second, numbers are used to count living beings, most importantly people. Ezra 2 gives a detailed listing of those returning from the Babylonian exile.

Revelation 7 gives a count of those who will be sealed as servants of God during the Great Tribulation.

Third, numbers represent spiritual qualities. The number 1 represents unity (Deuteronomy 6:4, Ephesians 4:3-6). The number 2 represents fellowship. The relationship between the Father and Son is the epitome of fellowship. This fellowship is exemplified in the relationship between Christ and the church, and a husband and wife. The number 3 represents community (Ecclesiastes 4:12, Matthew 18:20). The natural 7 represents completion or perfection (Genesis 1, Revelation 1:4,20; 5:1,6; 8:2; 10:3; 15:7).

The Bible, in addition to its many references to the natural numbers, does refer to the rational numbers (Genesis 28:22; Luke 19:8a,) and even, indirectly, to the irrational number p (I Kings 7:23). The Bible provides us with an indication that God is the creator of some mathematical objects, namely the integers, rational numbers, and real numbers. And from passages like Colossians 1:15-16 we can conclude that the entire trinity – Father, Son, and Holy Spirit – have participated and do participate in the creation/sustaining of all of creation, including mathematics:

> He [that is Jesus Christ, God the Son] is the image of the invisible God, the firstborn over all creation. For by him all things were created: things in heaven and on earth, visible and invisible, whether thrones or powers or rulers or authorities; all things were created by him and for him.

We are left with a philosophy of mathematics that provides for the possibility of mathematical objects proceeding from the mind of God, and thus having been created from all eternity.

Menzel (see Howell & Bradley, 2001) proposes a distinctly Christian view of mathematics, building upon Augustine's doctrine of divine ideas, which will be referred to as *creationism*. Creationism has four essential presuppositions (in addition to being theistic), providing distinction and clarification when comparing and contrasting with other philosophies of mathematics.

First, creationism maintains that God's creative work in *continuous*. God sustains His creation in such a way that it is continuously dependant on Him. Paul, in his letter to the Colossians, continues: "He is before all things, and in him all things hold together" (v.17). Second, creationism is *activist*. God's creation is a result of His divine mental activity. In other words, objects are created because God thinks them into being. God spoke "Let there be…" in the six days of creation. The Biblical narrative implies that God was speaking as humans speak – what is "on their mind." However, when God speaks it necessarily becomes reality. Third, creationism is *abstract object inclusive*. As a general rule, theists would include mental objects and spiritual objects in the list of God's invisible creation. Creationism also includes abstract objects like propositions, relations, and universals in this list of God's invisible creation. Fourth, creationism is *mathematically inclusive*. Mathematical objects, like numbers and sets, are believed to be created by God, or more accurately, since they seem to have an eternal quality, proceed from the mind of God.

Menzel addresses some objections to creationism, in particular the inclusion of abstract objects in God's creation. The first difficulty is with God's uniqueness: If abstract objects are eternal then God, it seems, is only one of many eternal beings. Menzel solves this problem by claiming that mathematics is divinely sustained by God, and so is not God's equal.

The second difficulty is with God's sovereignty: If abstract objects are necessary, then God, it seems, is just one of many necessary beings. Menzel solves this problem by appealing to continuous creation. Abstract objects exist necessarily because God necessarily sustains them. The third difficulty is with God's freedom: If abstract objects are necessary, then God's freedom, it seems, is limited. Menzel solves this problem by appealing to activism. God creates and sustains abstract objects through His mental activities. Abstract objects are necessary because they are created based upon God's very nature, and God cannot act contrary to His own nature.

Creationism, therefore, provides for the possibility of mathematical objects proceeding from the mind of God, and thus having been created from all eternity. Mathematical objects, like numbers and sets, are believed to be created by God, or more accurately, since they seem to have an eternal quality, proceed from the mind of God.

<u>Premise 6: Any model of a first-order theory exists in the objective sense</u>. The God of the Bible is omnipotent and He therefore must have access to any methods for constructing mathematical objects, including first-order theories. This does not mean that God actually uses first-order theories when constructing mathematical objects, but that his constructions would at least include first-order objects. Nor does this mean that God may not have other methods for constructing mathematical objects that have never been discovered. An important first-order theory is called Zermelo-Fraenkel set theory (see Jech, 2002), with the following axioms:

Axiom in ZFC	**Description**
Extension	If two sets have the same members, then they are the same set.
Null Set	The empty set exists.
Pairs	For any two sets, X and Y, there exists a set {X,Y} that contain exactly X and Y.
Unions	For any X there exists a set $Y= \bigcup X = \{z: z \in x \text{ for some } x \in X\}$, the union of all elements of X.
Power Set	For any X there exists a set $Y = P(X) = \{Z: Z \subseteq X\}$, the set of all subsets of X.
Infinity	There is a set X which contains ∅ and whenever $y \in X$ then $y \cup \{y\}$ is also a member.
Regularity	Every nonempty set has an $\in$-minimal element, i.e. no set is a member of itself.
Replacement Schema	If f is a function, then for any X there exists a set $Y=f[X] = \{f(x): x \in X\}$.

Separation Schema If φ is a property (with parameter p), then for any X and p there exists a set $Y = \{u \in X: \varphi(u,p)\}$, that contains all those $u \in X$ that have property φ.

Some reflection on these axioms is in order. First, it is remarkable that most (all?) of mathematics uses these nine axioms, albeit implicitly, and so these axioms must be true of all mathematics. Second, these axioms seem to be necessary conditions for the construction of any set. In other words, any set that God creates must have these properties. Now these properties cannot be external bounds by which God must operate, because then there would be a law higher than Him. Therefore these eight axioms must be aspects of God's nature.

Most set theorists would also include the following Axiom of Choice: Every family of nonempty sets has a choice function. That is, given a collection of sets, one can always pick an element from each set. The axiom of choice can be shown to be independent of the other axioms, and its negation can be taken as an axiom without losing consistency. In other words, one does not have any evidence from logic itself as to whether this axiom is needed. However, certain properties, like the well-ordering principle, are dependent on this axiom and justify its inclusion in set theory.

Although Zermelo-Fraenkel set theory with the Axiom of Choice (ZFC) does not provide for a reasonable apparatus for proof, at least not initially, it does provide a nice foundation to specify exactly what properties sets should have and how set operations should behave. In other words, sets should allow for all the common operations that we take for granted like union, intersection, complement, power set, and equivalence. Set theory can be viewed as a mechanism by which God thinks mathematics into existence. The bold conclusion from ZFC, since it is a first-order theory, is that the following objects must exist and proceed from the mind of God:

i) Natural Numbers
ii) Integers
iii) Rational Numbers
iv) Real Numbers
v) Complex Numbers
vi) Quaternions
vii) Non-standard models of the real numbers
viii) Multidimensional spaces from previous objects
ix) Function spaces of previous objects
x) Operators on functions of previous objects

From this list and logical extensions we can include almost every mathematical set studied by mathematicians. Also from ZFC we are led to conclude that sets are formed in stages.

> For each stage S, there are certain stages which are *before* S. At stage S, each collection consisting of sets formed at stages before S is formed into a set. There are no sets other than the sets which are formed at the stages (Schoenfield, 1977, p.323).

Menzel (2001) appeals to transfinite induction to also include properties, relations, and propositions (PRP's) in a similar stage development. God is able to create mathematics in logical, not necessarily temporal, stages all the way through the Cantorian infinite.

Stage 1. Concrete objects. Simple properties and relations.

Stage □, a non-limit ordinal. Everything in the previous stage. Sets formed from objects in the previous stage. New properties, relations, and propositions logically deduced from the previous stage.

Stage □, a limit ordinal. Everything in all previous stages. Sets formed from objects in all previous stages. New properties, relations, and propositions logically deduced from objects at all previous stages.

Because God has access to set theory, we can conclude that mathematical objects and associated PRP's were created and are sustained by the God of the Bible, and that these objects actually proceed from the mind of God. However, we should avoid concluding that set theory is the very method that God used to create mathematics. Schoenfield (1977, p. 324) cautions that:

> It is, of course, possible that there is a completely different analysis of the notion of a set, and this might lead to quite a different set of axioms. Up to the present, however, there has been no analysis of the notion of a set essentially different from that given here which leads to a satisfactory system of axioms.

3. Creationism and other mathematics philosophies

Up until now we have built up a philosophy of mathematics, by its bootstraps, so to speak. However, mathematical philosophy can never be done in an historical vacuum. There are other competing philosophies of mathematics, which mathematicians have held throughout history. Some of them are encapsulated in the following eight claims.

A. Mathematics is how I interpret it in my mind.

B. Mathematics is a social phenomenon.

C. Mathematics exists in physical objects.

D. Mathematics is observation of the physical/logical world.

E. Mathematics appeals to intuition.

F. Mathematics follows the rules of logic.

G. Mathematics is abstract symbols.

H. Mathematics exists independently from physical reality.

Loosely speaking, philosophies of mathematics can be broken into two camps, *skepticism* (illustrated by claims A-D) and *foundationalism* (illustrated by claims E-H). Skepticism argues that one can never hope to know whether mathematics is true and thus must accept mathematics as only a model of reality. Skeptics differ as to what mathematics is modeling. Foundationalism posits the existence of a method for putting mathematics on an irrefutable basis, hence allowing one to conclude that mathematics must be true. Foundationalists differ as to what constitutes this irrefutable basis. The various philosophies of mathematics, as illustrated by claims A-H, can be placed upon a continuum based upon the amount of belief or doubt in the objective reality of mathematics.

Doubt ◄────────────────────────────────► Belief

SKEPTICAL PHILOSOPHIES FOUNDATIONALIST PHILOSOPHIES

Constructivism Culturalism Materialism Empiricism || Intuitionism Formalism Logicism Platonism

Constructivism argues that mathematics is created in the mental constructions of the mind as one interacts with the physical world. Constructivism argues that understanding mathematics can never be separated from the mental constructions that one has of the physical universe and of the concepts of mathematics. Bransford, et. al. (1996) have demonstrated that personal constructions help to explain how mathematics is learned and how the learning progresses through various stages of abstraction. The mental images that one has for various mathematical objects do influence one's ability to communicate mathematics to others and the methods that one uses to solve problems. However, constructivism alone cannot account for the fact that definitions and theorems in mathematics are presented as facts to be understood by the reader, and not dependent on the individual constructions that readers might have.

Culturalism argues that mathematics is purely a cultural phenomenon, like literature and art, dependent on various times and locations. Grabiner (1974), in comparing 18^{th} verses 19^{th} century mathematics, has argued that the standards for mathematical truth have changed over time. Teachers of mathematics history courses are well aware that teaching mathematics in a cultural setting helps students to appreciate what provided the impetus for the development of various mathematical fields and to view mathematics as a human endeavor. Christian teachers can also point out the importance of faith in the lives of many mathematicians, for example Gottfried Liebniz and Georg Reimann, the founders of Calculus. However, Culturalism cannot account for the mathematics that seems to transcend culture. Clear examples of this include number, operations (addition, subtraction, multiplication, division), and the univalence principle of functions (knowledge of A informs about B). These mathematical concepts appear not to require human culture for verification.

Materialism argues that mathematics only exists as a "scientific matter capable of scientific justification" and not just a "whim or an aesthetic preference" (Maddy, 1998, p. 380). Loosely speaking, the materialist believes that mathematics exists in material objects. Materialism accounts for the fact that at every level of observation – from Hydrogen atoms, to DNA, to the earth, to the Milky Way galaxy – everything seems to behave with certain mathematical consistency. One cannot escape that the universe is mathematical. However, materialism does not account for aspects of mathematics that are not simply physical. Since mathematics in some sense models mental activity, must we conclude that the mind is purely a physical entity? Is the number 2 or mathematical functions or a derivative in Calculus simply a physical entity? Aren't some mathematical objects more like other abstract ideas including emotion, pain, and morality? And finally, materialism cannot account for the practice of mathematics, since mathematicians rarely discuss or use mathematics as if it were purely physical.

Empiricism argues that mathematics is observation of the physical universe. Lakatos (in Tomoczko, 1998, p. 30) argues that mathematics is "the results of bold speculation that have survived the test of severe criticism." Empiricm, though similar to materialism, is more liberal in its interpretation of "real" mathematics. Mathematics that works – adequately describes physical phenomenon – is admissible, even if the mathematical assumptions cannot be externally verified. Mathematical patterns are all around us: the motion of planets, the number of leaves on a tree, the continuation of time. In fact our understanding of the physical world would be incomplete without mathematics. Furthermore, axioms of mathematics are sometimes chosen to be consistent with observational data. However, empiricism cannot account for much of mathematics that is more than just observation. What is one "observing" when studying infinite dimensional space, group theory, or the Mandelbrot set?

Intuitionism, as defended by Brouwer (1981), argues that mathematics can be verified because it appeals to everyone's intuition. Intuitionism accounts for the concept of number, which we experience almost continually. Algebra, Geometry, and most of Calculus may seem fairly intuitive at first glance. Teachers of mathematics often note that the intuitive feel of mathematics should never be lost, and many teachers attempt to begin with the knowledge base of the student. But intuitionism fails to account for much of mathematics that is counter-intuitive, as illustrated when one studies surfaces of revolution. Objects in three-space can have infinite length, but finite area, a very unexpected and surprising result. Intuition, in this case, conflicts with the logical conclusions of Calculus. To move forward the mathematician must believe the Calculus and suspend initial intuitive expectations. Doing Calculus, in fact, causes a change in one's intuition, not the other way around. Intuitionism also cannot settle a debate when what is intuitive to one person is utterly paradoxical to another.

Logicism, as defended by Russell (1963), claims that mathematics is a subset of logic, which theorems proceed from undefined terms and assumed axioms. Logicism emphasizes the consistency with which mathematics follows the rules of logic and which sets mathematics apart from other disciplines. The development of axiomatic systems and the ability of mathematicians to write down rigorous proofs for so many mathematics facts are taken as demonstrations of the truth of logicism. However, logicism cannot account for aspects of mathematics that involve more than just logic: the teaching of mathematics, the process of deciding how to prove something, and the determination of what mathematics is worth studying.

Formalism, as defended by Hilbert, claims that mathematics can be completely described in abstract symbols and that mathematics is nothing more than the rules of a particular mental game. He was noted for the comment: "Instead of points, lines and planes, one must be able to say at all times tables, chairs, and beer mugs" (Kay, 1977, p. 79). Formalism understands the goal of mathematics to be the creation of a universal mathematics language so that everyone can understand mathematics regardless of one's native language. Formalism accounts for abstract symbols that help to perform calculations fluidly. Algebra is a great example of a symbol system that allows for quick calculations. Symbol systems like predication calculus, for instance, have revolutionized mathematics. However, reliance on abstract symbols alone results in the following problems. Abstract symbols sometimes obfuscate definitions that can be more simply stated with spoken language. Abstract symbols hinder laypersons from grasping mathematical concepts. Performing proofs exclusively using abstract symbols is extremely taxing and difficult, and in fact is sometimes impossible.

Platonism claims that "mathematical concepts and truth inhabit an actual world of their own that is timeless and without physical location" (Penrose, 1994, p. 50). Mathematical forms, or the Platonic world, thus exist independently of both the physical universe and mental constructions. Moreover, Penrose, in line with Plato's own philosophy, views both the physical world and the mental world as shadows of the necessary, eternal Platonic world. However, Platonism is unable to account for the fact that mathematics, as part of the Platonic world, seems to emerge mysteriously from the mental world. Platonism alone cannot determine whether or not our understandings of mathematics correspond to the true mathematical objects themselves. Moreover, Platonism cannot account for the existence of the eternal Platonic world and may lead some, like the Pythagoreans, to mathematical idolatry, with the Platonic world usurping the ontological role of God.

Without creationism, the mathematician is left with essentially eight competing perspectives or philosophies of mathematics, each which seem to have something useful to say about mathematics, but each which alone cannot account for all that mathematics is about. Furthermore, none of these philosophies relate to a Christian's most important source of understanding: the general and special revelation of God. Without a moral component, there is no mechanism to decide which mathematical philosophies to accept as true. So creationism is needed to provide the Christian mathematician with a perspective in order to answer many mathematical questions, not the least of which is whether anything in mathematics exists at all in the objective sense.

Having described various mathematical philosophies, the creationist can begin to critique them. The following table illustrates how creationism agrees and disagrees with other philosophies of mathematics.

Philosophy	Agreement	Disagreement
Constructivism	Mathematics is grounded in one's own constructions	Idealized constructions of mathematics are identified with divine constructions.
Culturalism	Cultures view mathematics differently because humans are not divine	There is an objective God's-eye view of mathematics.
Materialism	Mathematical objects are like their physical counterparts because they have a common creator.	Mathematical objects are independent of physical reality.
Empiricism	Mathematical objects describe their physical counterparts because that is how God describes them	Mathematical objects also concern the mind of God.

Intuitionism	Mathematics makes sense because humans are made in God's image.	Mathematics is about what God, not people, deems intuitive.
Formalism	Abstract symbols describe the language of mathematics.	Mathematics is God's reality, not simply the rules of a game.
Logicism	Mathematics follows the rules of logic because God is logical.	God, not proof, establishes the truth of a statement.
Platonism	Mathematical form does have its own existence.	Mathematical existence is totally dependent on God.

In relationship to skeptical philosophies, Creationism does recognize that mathematical learning is mediated through the human mind and physical reality, but denies that mathematics is simply an observational science. In relationship to foundational philosophies, Creationism upholds the irrefutable basis of mathematics, but denies that such a foundation can be found apart from God, either as a separate reality or through logic or symbols or intuition.

4. Conclusions

Mathematicians must, at some point, account for the value of their work, or be drawn to some other more meaningful activity. A mathematician who values mathematics finds satisfaction and is motivated to continue intellectual pursuits despite difficulties. Christian mathematicians, even more so, must place the value of their work in the context of Jesus' command to:

> Therefore go and make disciples of all nations, baptizing them in the name of the Father and of the Son and of the Holy Spirit, and teaching them to obey everything I have commanded you. (Matthew 28:19-20a).

Consequently, mathematicians of faith need a philosophy robust enough for their discipline to either uphold the value of mathematics, or else to guide them toward other pursuits.

This paper has demonstrated the viability of creationism as a philosophy of mathematics. Creationism argues that mathematical objects, like other eternal abstract objects, proceed from the mind of God. Creationism and first-order logic provide a mechanism to describe how mathematics is created and to defend the existence of certain mathematical objects including number systems, functions, and operations. Creationism gives the mathematician confidence that many of the objects of mathematics proceed from God's divine intellect. In fact, both God the Father and God the Son participate and have participated in the development of much of mathematics.

In the future, creationism might be useful in answering questions in mathematics that have eluded all attempts at formal proof. One specific example is the hypothesis that non-deterministic computability in polynomial time implies deterministic computability in polynomial time (the P = NP conjecture). Creationism would seem to respond to this conjecture in the negative, because God as a free agent is not bound by deterministic methods. Further investigation is needed.

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