

Augustine's Mathematical Realism

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Let me begin by giving a descriptive statement about what I take “Mathematical Realism” to be. Some philosophers of mathematics may identify the view as “mathematical Platonism.” I prefer the terminology “mathematical realism” as to avoid the prejudicial connotations which some might have regarding views held by Greek philosophers.

Mathematical Realism is the view that certain mathematical statements are true because they successfully refer to entities which exist independently of the human mind and describe relationships which indeed, do exist between them. We, humans, can discover such true relationships between these entities.

Some implications which are often drawn from this view are as follows:

1. In ontology and metaphysics: There are non-material objects.
2. In Philosophy of Mind and in Epistemology: The human mind has some kind of primitive intuition which accounts for the ability of humans to discover relationships between these non-material objects.

I am sure that you are all aware of the fact that mathematical realism is not getting a very good “press” these days. It is true that those engaged in the practice of mathematics favor realism; at least psychologically. But many mathematicians retreat to formalism when they are asked to give an account of the nature of the objects under consideration in mathematics.

But what is the case against mathematical realism? I shall summarize. The case has been given many times in the philosophical literature. An especially good example is the work of Philip Kitcher. He tries to show what tensions are involved in realism in a paper entitled *The Plight of the Platonist* which appeared in the journal *Nous* Volume 12, 1978, pages 119-134. I shall list the statements which Kitcher gives and then we shall provide further explanation of what is meant by each.

1. *There are true arithmetical statements.*

We are hereby asserting that there are some statements in arithmetic containing terms that successfully refer to objects and to relations between them and that these statements say of “what is” that “it is.” The tacit theory of truth that is involved is the correspondence theory of truth. In it we assume that there is an objective world outside of and independent of us humans and that to say the truth about this world is to correctly describe what is the case about objects in this world.

Notice that we are also saying that there are some true *arithmetical* statements; that is, there are some statements in arithmetic that are true. Examples which readily come to

mind are statements like

- a. $3 + 7 = 10$
- b. All natural numbers are either even or odd.

To summarize, we are saying that there are some arithmetic statements like the above which successfully refer to objects independent of us humans and which describe correctly what is the case about them.

2. We humans can know some of these arithmetical statements.

What we are saying here is that there are some true arithmetical statements that we humans can know to be true; that is we are so constituted and equipped that we can know some of these statements. Philosophers have written extensively about what constitutes knowledge. Many definitions that have been proposed have the following form.

A person *a* knows a proposition *p* just in case that:

- a. *p* is true.
- b. *a* believes the proposition *p*.
- c. *a*'s belief in *p* is warranted.

This third condition is most controversial and hence varies from one theorist to another. These conditions can be summarized as follows: a careful description is made of the conditions under which *a*'s belief that *p* is justified or warranted. The third condition is, of course, necessary because a person might believe a true proposition but for the wrong reasons and hence not know the proposition.

Mathematical Realists give an account of mathematical knowledge as follows. We human beings have an intuition which accounts for the possibility of our mathematical knowledge. This intuition may be activated by our early manipulations with physical objects. This intuition develops in human beings in parallel with their reasoning and deductive skills so that, guided by this intuition, and correctly using our reasoning skills we are able to find out the truth about arithmetical objects. The manipulation of physical objects precedes the idealization, and generalization, and abstraction which provides access to the objects of arithmetic.

3. The arithmetical statements which we humans know are useful in our study of and in our dealings with medium-sized objects and in our scientific work.

Mathematical Realists do have some difficulty in explaining the power of mathematics in its applications to the every-day world. In explaining the applications of arithmetic they might say that the fact that mathematical intuition is activated by everyday experiences with medium-sized objects accounts for the amazing usefulness of arithmetic. Somehow, the rules which govern relationships which such objects bear to one another, are sufficiently like the laws which govern the relationships between

mathematical objects discovered using our mathematical intuition that we can apply our mathematical knowledge to them.

4. *A statement can be true only if the singular terms which it contains refer to objects.*

Again, the correspondence theory of truth is assumed here. The sentences which we humans utter which assert that something is the case; that is which are statements, contain singular terms like nouns and pronouns which must successfully refer to some entities in order to properly qualify as true or false. If such singular terms did not satisfactorily refer to some specific objects, we could not know whether such statements were true or false. For example, if we said that “He is going to the city today” and we did not know to whom the word “he” refers or if we did not know to which city the word “city” refers, we could not classify the statement as true or false, nor could we know that such a statement is the case.

Presumably, then, if we can know certain arithmetical statements, then it must be the case that the singular terms in such a statement must successfully refer to some specific entities or other.

5. *Many true arithmetical statements which we know contain singular terms which are numerals.*

Presumably, the arithmetic statements which are readily given as examples of arithmetic statements which we know contain the numerals 1,2,3,... and the like as singular terms.

6. *Thus, the numerals refer to objects of some kind.*

7. *But, numerals cannot refer to material objects because there are so many of them and also because the truth of arithmetical statements does not depend upon the fate of any material objects.*

Here we assume that the number of distinct numerals is (potentially infinite) and perhaps also that the number of material objects in the universe is finite. Also, it is difficult to imagine that the truth of arithmetic statements is dependent upon the “fate” of a material object since our experience with material objects is that they are transitory while our statements about numbers seem to be independent of time and space.

8. *So the referents of numerals (numbers) must be abstract, non-material objects.*

9. *If the numerals refer to objects, then there are particular objects to which they refer.*

10. *The objects to which the numerals refer are sets.*

This statement has an illustrious historical context. For centuries, it has been a tantalizing question for philosophers to explicate the concept of number. But during the early part of the twentieth century some fine pioneering work was done to do so. The German philosopher, Gottlob Frege, and later, the Englishman, Bertrand Russell, made very important contributions to the clarification of the number concept. Frege identified a number as a property of properties. Thus the number 2 is the property which a property

has just in case there are exactly two objects which have the given property. Thus

$$2(f) \equiv \exists x \exists y (f(x) \wedge f(y)) \wedge (\exists z)(f(z) \equiv ((z = x) \wedge (z = y))).$$

Put in English, the property f has the property “two” if and only if there are exactly two objects which have the property. The charge that the definition seems to be circular is erred defused by pointing out that the definition can be phrased in purely logical terms.

Bertrand Russell used set-theoretic terms to explicate the concept of number. He described the number two, for example, as “the class of all sets which are pairs”; that is the class of all sets which can be put in one-to one correspondence with some paradigm set, say $\{a, b\}$. Thus, the number 2 gets identified with all the class of all two-element sets.

Since the time of Frege and Russell, there has been much theoretical work in *set theory*. In such theories, there is usually no attempt to identify any objects for consideration except for sets themselves. Thus, the main concern in such theories is to describe operations under which sets can be constructed from previously given sets. Nevertheless, most set theories provide for the existence of an empty set which has no members. This means that many sets can be constructed using say, the operations of union and intersection, from this empty set. Furthermore, in many set theories certain sets are specified which are taken to play the role of the numbers 0,1,2,3, in the given theory. Thus, if \emptyset is the symbol for the empty set, then

$$\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \dots$$

are playing the roles of

$$0, 1, 2, \dots$$

within the theory.

So, the preceding statement refers to the fact that all modern explications of the concept of number identify numbers as sets of certain kinds. Perhaps the reason that Frege’s use of properties of properties gives ontological status to such a wide variety of non-material objects like properties and properties of properties, and then properties of properties of properties and that such ontologies may be subject to a snarl of logical difficulties force modern philosophers to limit considerations to formal, first-order, theories about sets.

11. *But there are no particular sets to which the numerals refer.*

It is a fact that many different set-theories have been proposed and that each has its own paradigm series of specific sets which play the role of the number series 0,1,2,3,... It also is clear that there are few good reasons for us humans to prefer one series of paradigms over any other and thereafter declare that our favorite is, in fact, the actual, one and only, list of entities to which the numerals refer. Again, it appears that there is no good reason to prefer one list of sets which is purported to be “the list” over any other.

Some philosophers have taken the method used by Russell to declare that the number “three”, for instance, is the class of all three-element sets. We are thus passing to equivalence classes which presumably contain the paradigms for the number three. Yet, others have said that the set-theoretic process of forming equivalence class using an equivalence relation involves functions and relations. Furthermore, the concept of function or relation involves the concept of *ordered pair* and, again, the definition of ordered pair varies from one set-theory to another and again there is no particular definition which is to be preferred over against another. Perhaps the most common way to define an ordered pair in set-theory is to say that

$$(x,y) = \{x, \{x,y\}\}.$$

Yet, there is no good reason that we humans can propose that will select any one way of defining an ordered pair over another.

12. *Thus the numerals refer to abstract objects which are not sets.*

The conclusion that we can draw is that the numerals that we humans have concocted do not successfully refer to any sets that we are aware of or that we can imagine will be of such a nature that we would prefer such over any other.

13. *There are no abstract objects distinct from sets.*

This statement is controversial. It says that there are two kinds of objects; material objects, and the abstract, non-material objects each of which is a set of some kind. Since the numerals of arithmetic do not refer to material objects and since the numerals of arithmetic do refer to non-material objects which are not sets, we are faced with contradiction. One cannot hold to all of the above statements 1-13 without being logically inconsistent. The mathematical realist is supposedly in a dilemma.

What can be said about Kitcher’s analysis? One is tempted to begin an analysis of these statements to show how to relieve the supposed tension. Certainly a theist will deny Statement 13 which asserts that there are no non-material objects distinct from sets! I have chosen not to refute the case against mathematical realism myself. Instead, I will reach back many centuries to the great church father, Augustine of Hippo to present the case for Mathematical Realism.

Before I describe Augustine’s views about mathematical knowledge, I would like to give a short description of Augustine’s life and times. I do so because it will provide a context for understanding his views.

The Life of Saint Augustine of Hippo

Saint Augustine, whose real name was Aurelius Augustinus, was born in Thagaste in northern Africa on August 28, 430 A. D. He played a key role in bringing about the transition from classical antiquity to the Middle Ages. He lived through the eighty years of the decline of the Roman Empire and he played a key role in the period of transition

from Roman paganism to Christianity.

Augustine's life is characterized as a life of transitions - transitions from one city to another, and transition from one philosophical or theological viewpoint to another, and from being under the influence of one important person to another. Such diverse influences are not to be viewed as indicative of a "helter-skelter" life for the great church father. His life can be viewed as a wonderful progression which finally arrives at resolution and completion. I shall describe some of these cities, some of the philosophical influences and also some of the important influential persons in his life.

Let me next describe some of the main aspects of Augustine's life. (Much of my summary is taken from the wonderful little book on the life of Augustine written by Peter Brown entitled *Augustine of Hippo*.)

Augustine received his early education in Thagaste and Madaurus. His early education was classical in nature, almost entirely literary. Augustine learned the art of words very thoroughly and he was trained to acquire the art of persuasion. His education placed a great premium on memory, was exclusively in Latin and had little philosophy, science, or history in it. Later on Augustine continued his education in the city of Carthage. It is here that he took a concubine and it is here that he also had a son whom he called "Adeodatus". It is here that he came under the influence of the Manichaeian sect and, in fact, he joined it for a short time.

Augustine was deeply influenced with the literary works of those who preceded him. Some of the books and pamphlets which influenced him most are these.

a. "Hortensius": a lost book of Cicero. "This book, indeed changed all my way of feeling. It changed my prayers to thee, O Lord; it gave me entirely different plans and aspirations. Suddenly, all empty hope for my career lost its appeal; and I was left with an unbelievable fire in my heart, desiring the deathless qualities of WISDOM, and I made a start to rise up and return to Thee... I was on fire, my God, on fire to fly away from earthly things to Thee." The Hortensius carried an exhortation to Wisdom for Augustine. (Confessions: III, iv, 7).

b. "Letter of Foundation": Mani. When the hearers of this letter heard the pronouncements of Mani, they were "filled with light". This was the basic religious experience of the Manichee; he was a man who had become acutely aware of his own state. Blessed is the man that shall know his own soul. He found that he could not identify himself only with a part of himself, his good soul. There are two opposite, equally real forces in the world which are locked in battle; namely good and evil and that each is represented in the life of a man. These Two Principles have natures which are absolutely distinct. As a Manichaeian he could grasp for himself the essence of religion.

c. In the dialogues of Cicero, *The Academici*, the skeptics are portrayed. According to the skeptics, the greatest virtue lays in suspended judgment. The greatest peril is in unheeding adherence to a single opinion.

d. The sermons of Ambrose at Milan. He later describes his reaction: "I noticed

repeatedly in the sermons of our bishop (Ambrose) that when God is thought of, our thoughts should dwell on no material reality whatsoever, nor in the case of the soul, which is the one thing in the universe that is closest to God.”

e. The *Enneads*: a discourse by Plotinus, an Egyptian Greek who had died in Rome close in 270. The *Enneads* were edited by a disciple of Plotinus, called Porphyry. In Milan, where Augustine lived for several crucial years of his life, much of articulate and fashionable Platonism was Christian. There was an audacious attempt to combine Platonism and Christianity. Perhaps Augustine read the short treatise by Plotinus called “On Beauty”. Augustine responded in the *Confessions*: VII, xvii, and 23. “For I wondered how it was that I could appreciate beauty in material things. . . and what it was that enabled me to make correct judgments about things that are subject to change, and to rule that one thing ought to be like this and another will like that. I wondered how it was that I was able to judge them in this way, and I realized that, above my own mind, which was liable to change, there was the never-changing true eternity of truth.” According to Plotinus “the outward-going diffusion of the One coincides with a continuous tension of every part to return to its source of consciousness. . .” The Neoplatonic doctrine includes “procession” outward and a “turning” inwards. The power of the good always maintains the initiative. The words of John in Chapter 1 of his gospel were taking on new meaning. “In the beginning was The Word and the Word was with God, and The Word was God.” Augustine was on his way to authentic Christianity.

f. The pagan Platonists regarded the Christian myth of Redemption, and Incarnation, a Crucifixion, and a Resurrection of the Body as a barbarous innovation on the authentic teachings of their master, Plotinus. Augustine turned to the sacred writings of Saint Paul. “Because even if a man takes pleasure in the law of God in his inner self,” what about the other law which I see in my members? What shall be done with “O wretched man that I am”? Who shall set him free? God was no lonely aristocrat; the therapy of God was made available to all men by an act of *popwaris elementia*. “Not in revelling and drunkenness, not in lust and wantonness, not in quarrels and rivalries. Rather, arm yourself with the Lord Jesus Christ and spend no more thought on nature and nature’s appetites.” Augustine later reports “I had no wish to read more and no need to do so. For in an instant, as I came to the end of the sentence, it was as though the light of confidence flooded into my heart and all the darkness of doubt was dispelled”. Augustine was baptized at Milan in the baptistery adjoining the basilica.

Augustine would devote the rest of his life to theology. In preparation, he retired to Cassiciacum to a country villa near Lake Como in the beautiful foothills of the Alps to write. He began a period of *Christianae vitae otium*, Christian cultured retirement. He proposed an appropriate training which would culminate in the contemplation of the Trinity. In his view, education was to be disciplined and, in order to be complete, it should be expanded to include the abstract sciences, geometry, and the mathematical background of astronomy. Augustine’s God is the God of the philosophers. He is the

founder of the harmonies of the universe and his relation to men is as absolute and necessary as the form of a geometrical theorem. Yet there is a sharp note of unrelieved anxiety about himself and of his dependency upon God.

Thereafter, Augustine lived in Ostia, where his mother, Monica, who had brought him up as a Christian, died. Later he lived in Rome and Carthage. Cultivating a few close friends he then returned to Thagaste with his *servi Dei*. These men were baptized, dedicated, laymen, determined to live in the company of bishops, priests and noble patrons, the full life of a Christian. Augustine wanted thereafter to be more than contemplative. Eventually he went to Hippo where he became a priest and then later the Bishop of Hippo. "Whoever thinks that in this mortal life a man may so disperse the mists of bodily and carnal imaginings as to possess the unclouded life of changeless truth, and to cleave to it with the unswerving loyalty of a spirit wholly estranged from the common ways of life - he understands neither what he seeks nor who seeks it." Augustine realizes that he will never reach the fulfillment that he had first thought was promised him by Christian Platonism.

Augustine eventually came to appreciate the sheer difficulty in achieving the ideal life. A looming fact is the permanence of evil in human actions. Again, it became obvious that the human will does not enjoy complete freedom. As St. Paul says the "flesh lusteth against the spirit and that which you wish not, that you do. . . Who shall deliver me. . ." Man's body is not only the tomb of his soul. In the final analysis, the lives of his parishioners became a book for Augustine to read. This leads Augustine to a new humility. Again, he turned to St. Paul. In June of 394, in giving lectures on the book of Romans, he began to see with Paul that in this life there is always unresolved tension between flesh and spirit. The true image of life is *inter*, a long highway. "To solve the question, I had previously tried hard to uphold the freedom of choice of the human will; but the grace of God had the upper hand. Paul said 'Who has made you different? What have you gotten that you did not receive? If you have received all of this, why glory in it as if you had not been given it?' " The notion of "delight" and feeling had taken its place as the ally of the intellect. Delight is discontinuous and startlingly erratic. All a man can do is to yearn for an absent perfection. *Desiderium sinus cordis*. It is yearning that makes the heart deep.

Augustine, the contemplative philosopher, became a priest. In the long run his intellectual interests became transformed by his new duties. Augustine deeply felt the need to slowly adjust to a life of authority at Hippo. "Darkness has overwhelmed me... Oh, that I had the wings of a dove, that I might flyaway and be at peace." Augustine had the attitude (in distinction from the Donatists) that the church had the power to absorb the world without losing its identity. The church is hungry for souls: let it eat, indiscriminately if it need be. The Donatist movement, on the other hand, was like the Ark of Noah. It was well-tarred, inside and out. Augustine's Neo-Platonism had profoundly affected his conception of the church. The rites of the church take on an objective and permanent validity, independent of the subjective qualities of those who participate in them.

"Whoever does not want to fear, let him probe his inner self. Do not just touch the surface; go down into yourself; reach into the farthest corner of your heart. Examine

it then with care: see there whether a poisoned vein of the wasting love of the world does not pulse, whether you are not moved by some physical desires . . . then only can you dare to announce that you are pure and crystal clear, when you have sifted everything in the deepest recesses of your inner being.” Sermon 348, 2

Let me point out some of the important aspects of Augustine’s life which will serve as a background for understanding his epistemological views.

1. Augustine was a key figure in the transition from Classical Antiquity to the Middle Ages; from a time when Greek philosophy and culture were predominant in the world to the time when Christianity had not only won the hearts of men but had also gained political ascendancy. Thus, Augustine was a man who stood tall in two worlds; the classical Greek world and the Christian world. Though he felt the tension and dissonance between them, he managed to combine them in very creative ways in his thinking.

2. Augustine was trained in rhetoric and in literature and not in the sciences and mathematics. His academic training placed a great deal of attention on the literary works of Virgil, Cicero, Sallust, and Terence. There was little emphasis upon science, philosophy and history. By training, therefore, Augustine was skilled in writing and in the art of persuasion. As an academic, he held professorships in rhetoric.

3. He was a prolific writer. By my count in our library, there were 26 volumes on the shelf in one collection of the translations of his writings. He wrote on many different topics from metaphysics, epistemology, and the philosophy of mind to theology and expositions of the Sacred Scriptures. His writing was not systematic and it is not hard to detect changes that he underwent in his thinking. He openly acknowledged such changes. His writings which were philosophical in nature were never cold and detached. His ideas were presented with great passion and eloquence and were directed towards the goal of happiness and blessedness for his readers.

4. Augustine was deeply influenced by the Scriptures in his writings. It is alleged that he refers to the scriptures some 42,817 times in his writings.

5. Augustine was a man who had deep need for love and affection from others. He was a man who was profoundly influenced by interpersonal relationships. I cite his strong attachment to his mother Monica and also his constant contact with male friends, especially Alypius and Nebridius. His writings are self-disclosing, indeed. They reveal his deepest thoughts and even darker side of his life. How otherwise would we know that he took a concubine for 15 years with whom he had a son, Theodatus? Again, he often reveals the sinful tendencies within him as a wielder of great power and influence in the latter stages of his life as Bishop of Hippo.

Now that we have had a glimpse of the life of Augustine, we shall turn to a general look at what I will call his “world and life view”. How did Augustine look at the

world? What is the general setting of his epistemological and ontological views? I shall include a diagram which nicely illustrates Augustine's cosmology. It is taken from the excellent book by Ronald Nash entitled *The Light of the Mind: St. Augustine's Theory of Knowledge* (The University Press of Kentucky: 1969, page 5). Notice the important triads in Augustine's thinking. As to his ontology he discerns three levels of becoming; the first and foremost being God, the source of all being; next souls, including the souls of men and angels; and finally, bodies, the material things of the world. As to his theory of knowledge, Augustine sees knowing as beginning in the senses and progressing to knowledge of the imagination, and, finally, culminating in the knowledge of the intellect or reason which knows the spiritual and the eternal. All of this is manifested in a duality which mankind possesses. Man has a *ratio inferior* which describes his knowledge of material things; his *scientia*. But, more importantly, mankind has a superior knowledge, *sapientia* or wisdom, by which he discerns the spiritual.

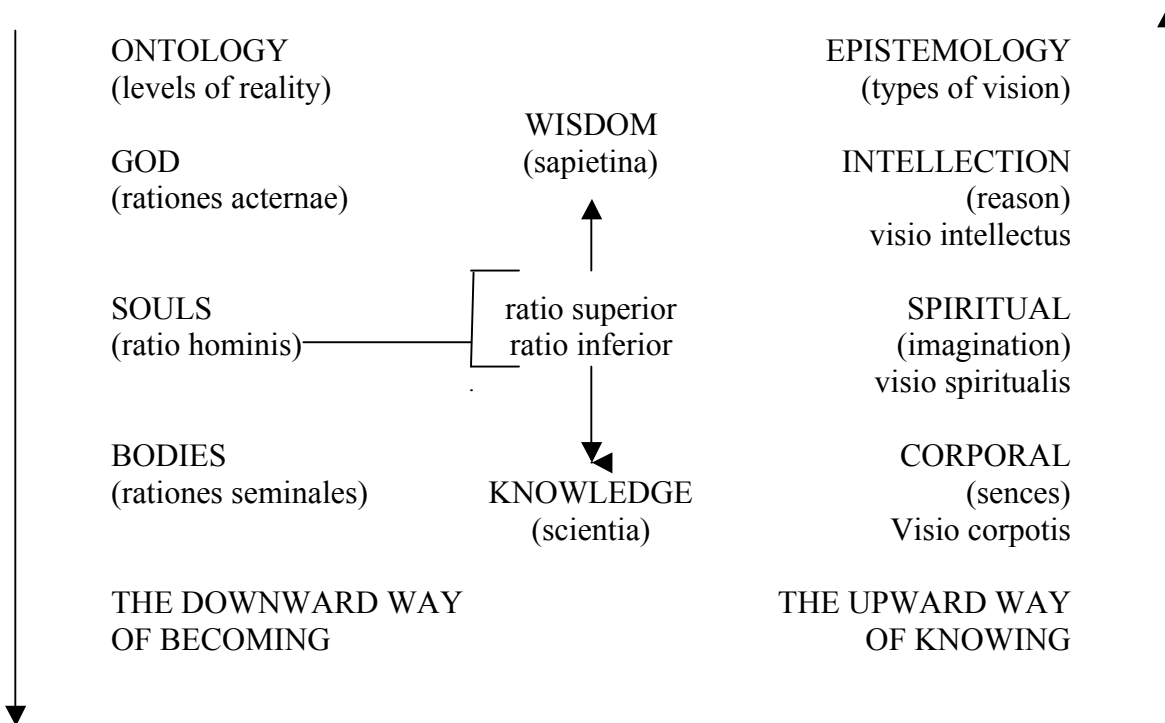
Before considering Augustine's views about mathematical knowledge, let us look a little more carefully at his general epistemology.

Augustine on Knowledge

Augustine squarely faces the Academics who questioned whether we humans can ever know anything for certain and who advocated a cultured tentativeness for themselves and for others as our best stance towards knowledge claims. The first part of his argument against the Academics is to identify one kind of knowledge which is certain; namely the self-knowledge of one's own existence. Augustine's argument is summarized in the epigram *Si fallor, sum*: If I doubt, then I am. Augustine hereby claims that we have privileged access to our own self-existence since doubting such is a proof of our existence.

Furthermore, Augustine continues, if Skepticism is true, then search for God is ruled out from the start and thus skepticism also leads to immorality. The first step along the path leading to the mind of God is to accept revelation by faith. The famous epigram *Crede ut intelligas* summarizes Augustine's views on the relation between faith and reason.

REASON AND KNOWLEDGE



Nash, Ronald, The Light of the Mind: St. Augustine's Theory of Knowledge, The University Press of Kentucky, 1969, page 5.

Augustine also comes to the defense of sense experience in his argument against the skeptics. He asserts that Christians “believe also that the evidence of the senses which the mind uses by aid of the body; for if one who trusts the senses is sometimes deceived, he is more wretchedly deceived who fancies that he should never trust them.” (Contra Academicos: 30, 111, 11, 25) In his defense of knowledge claims involving the senses, Augustine relies heavily on notions about the memory (memoria) and the accompanying capacity of the mind to construct images. To Augustine, an image in the memory plays a role in all knowledge claims. Such memory is not accounted for by “anamnesis”, which is a recollection from a past existence, as asserted by Plato. Rather as Etienne Gilson says. “The Platonic recollection of the past gives way to the Augustinian memory of the present.” To Augustine, knowledge claims about the past in which we assert recollections from previous experiences which are images constructed by the mind are essentially not different from the immediate assertions which we make about our everyday experiences. Furthermore, the processes which account for such knowledge claims are INNER. To remember is attend to an image which is immediate to the INNER eye.

Gilson describes the view of Augustine as follows. “Once an image of the present

corporeal object is formed by the spiritual vision, the inner man judges that object by appeal to the eternal truths that are provided by illumination.” Non-material, eternal, extramental objects permit us to judge material, temporal extramental objects. Gilson says, “So, contrary to all expectations, the analysis of sensation has brought us back from the exterior of things to the interior of the soul.” From the material object we get the image in bodily sight from which we get an image in the memory from which we get an image in the gaze of thought.

As an example, in his essay *De Musica* in which Augustine analyzes how we come to recognize the meter in the Strabose in the hymn *Deus Creator Omnium*, there are described five stages in the process of recognition. There is the voiced stage (the production of sound waves), the sounding change, the noise stage, the remembered stage, the discriminating stage.

What is emerging in this description of Augustine’s views is the vital role played by the “inner man,” even in the use of his senses to know about material things.

The part of Augustine’s theories that is of most interest to us as mathematicians is his theory of a priori knowledge. Of course a prior question to ask is whether there is any such knowledge as a priori knowledge; that is knowledge which is independent of experience and which is certain. Another associated question is whether there are such entities as objects of a priori knowledge. Augustine’s answer is a resounding “yes” in both cases and God plays a vital role in Augustine’s account for each.

Interestingly, Augustine begins his argument by stating that the claims which we make about numbers as excellent examples of a priori knowledge. Such knowledge is universally the same for all men. It is certain, and is about “principal ideas” which exist in the mind of God. It is God, therefore, who is the explanation of our knowledge about numbers and other mathematical objects. In giving this account of our knowledge of mathematical objects, Augustine rejects the Aristotelian view that “numbers make their impression on our minds not in their own right, but as images of visual things which spring from our contacts with corporeal object.” We do not merely abstract the concept of number by experience with corporeal objects. Let me quote from the essay *De libero arbitrio* where Augustine explains his view at length. The dialogue is between Evodius and Augustine. (On Free Will: Book II,20)

Aug: Now consider carefully, and tell me whether anything can be found which all reasoning beings can see in common, each with his own mind and reason; something which is present for all to see but which is not transformed like food and drink for the use of those for whom it is present; something which remains complete and unchanged; whether they see it or do not see it. Do you perhaps think there is nothing of that kind?

Ev: Indeed, I see many such, but it will be sufficient to mention one. The science of numbers is there for all reasoning persons, so that all calculators may try to learn it, each with his own reason and intelligence. One can do it easily, another with difficulty, another cannot do it at all. But the science itself remains the same for everybody who can learn it, nor is it converted into something consumed like food by him who learns it. If anyone makes a mistake in numbers, the science itself is not at fault. It remains true and

entire. The error of the poor arithmetician is all the greater, the less he knows of the science.

Aug: Quite right. I see you are not untaught in these matters, and so have quickly found a reply. But suppose someone said that numbers make their impression on our mind not in their own right, but rather as images of visible things, springing from our contacts by bodily sense with corporeal objects, what would you reply? Would you agree?

Ev: I could never agree to that. Even if I did perceive numbers with the bodily senses I could not in any way perceive their divisions and relations. By referring to these mental operations I show anyone to be wrong in his counting who gives a wrong answer when he adds or subtracts. Moreover, all that I contact with a bodily sense ... I do not know how long it will last. But seven and three make ten not only now but always. In no circumstances have seven and three made anything else other than ten, and they never will. So, I maintain that the unchanging science of numbers is common to me and to every reasoning being.

Augustine argues that numbers are not conveyed to us by the bodily senses. His argument is that whatever we contact with a bodily sense is proved to be not one but many. "But however I come to know unity, I have not learned it from bodily senses. . ."

In section 23, Augustine argues that we know the truth of the firm, unbroken rule that the fourth number after 4 is its double, the fifth number after 5 is its double, . . . the double of any number is found to be exactly as far from that number as that number is from the beginning of the series. He then says: "How do we find this changeless, firm and unbroken rule persisting throughout the numerical series? No bodily sense makes contact with all numbers, for they are innumerable. How do we know that this rule holds throughout? How can any phantasy or phantasm yield such certain truth about numbers which are innumerable? We must know this by the inner light, of which bodily sense knows nothing. By many such evidences all disputants to whom God has given ability and who are not clouded by obstinacy, are driven to admit that the science of numbers does not pertain to bodily sense, but stands sure and unchangeable, the common possession of all reasoning beings... But I am glad that the science of numbers most readily occurred to you when you had to answer my question. For it is not in vain that the holy books conjoin number and wisdom, where it is written: 'I turned and inclined my heart to know and consider and seek wisdom and number.'" (Eccl 7:25).

In question 46 of *De diversis Quaestionibus: LXXXIII, De Ideis* Augustine says, "...the principal ideas are certain images, or stable and unchangeable reasons of things, which not being themselves formed are consequently eternal and always the same, and are contained in the Divine intelligence."

To see Augustine's view of the role played by "principal ideas" in perception consider the following quotation from Augustine. It is found in *De Trinitate: XI,6,10*.

"...the images of corporeal things ... which we draw in through the bodily sense and which flow in some way into the memory, and from which things that have

not been seen are also presented to the mind under a fancied image, whether it contradicts the reality or by chance agrees with it, are approved or disapproved within ourselves by rules that are wholly different, which remain unchanged above our mind when we rightly approve or disapprove of anything.”

In Augustine’s view, therefore, the eternal ideas that govern the universe must be God’s Divine ideas. God uses these principal ideas as templates in his creation of material objects. Mankind has access to these Divine ideas in his inner self. Furthermore, in the gaining our knowledge through our senses, the principal ideas that we possess in our inner selves play an integral part in the judgments which we make. Bruce Bubacz has described the Augustine view as follows.

He says that “in the dual functioning of *visio spiritualis* we can see most clearly Augustine’s notion of a human being as a unity of the immaterial spiritual realm and the corporeal, temporal realm.” He describes the interaction of the inner man and the data which it receives from the physical world as “vital attention.” How does this vital attention operate, according to Augustine? Bubacz describes the process as follows. *Visio corporis* presents unorganized raw sensations to the inner man. Using clues (in particular those clues associated with the preservation of the life of the outer man) the inner man constructs an image representing a likely coherent systematic picture of the physical world. Since the physical world was constructed by God in accord with His divine ideas, the physical world is formally harmonious - it makes sense. In order to represent this world accurately, the inner man must construct a picture of the world that itself is harmonious. In making this picture, the inner man is illuminated by the principal ideas which were placed there by God. Among the principal ideas which men possess are his ideas about numbers and other mathematical objects.

Of greater concern to Augustine than universals like mathematical objects are the more important universals like truth, and justice, and wisdom, and the beautiful. He says, “Concerning universals of which we have knowledge, we do not listen to anyone speaking and making sounds outside themselves. We listen to truth which presides over our minds within us . . . Our real Teacher is he who is so listened to, who is said to dwell in the inner man, namely Christ, who is the unchangeable power and wisdom of God.”(*De Trinitate:XI*)

In order to explain how we humans gain knowledge of these essential universals Augustine uses the analogy between physical and mental sight. He says, “But we ought to believe that the intellectual mind is so formed in its nature as to see things, which by the disposition of the Creator are subjoined to things intelligible in a natural order, by a sort of incorporeal light of a unique kind; as the eye of the flesh sees things adjacent to itself in this bodily light, of which it is made to be receptive and adapted to it.” We “see” the principal ideas as objects in the same way that we see physical objects. To see a physical object, we need not only an observer and something to be observed. We also need light of illumination which falls upon the object to be observed and an observer which is attentive to the situation. In like manner, in order to acquire knowledge about numbers and, more importantly, about truth and beauty and justice we need illumination

which dwells in the inner man and whose presence is ultimately due to God Himself.

Let us now return to the first part of my talk which describes the tensions which are supposedly present when one holds to mathematical realism. Augustine's defense would at least contain the following points. First, it is nonsense to say that the only nonmaterial entities that exist are sets. The list of nonmaterial entities is much richer than that! Secondly and more important, is the reality that God exists and that it is He who is the ultimate reason behind the existence of all else. This God that we believe in is a personal God with whom we have to do. It is He who has made each of us in His Image. It is He who has equipped us for knowledge. It is He who provides the inner illumination so that we can know justice and truth and beauty. It is He who illuminates our thoughts about numbers and their properties even though we may not be aware of his presence.

I am sure that all of this sounds very strange to our twentieth century, Western ears. Mathematical Realism is not a serious choice for many philosophers today. For examples one has only to think of the almost gleeful conclusions which Morris Kline draws at the end of his book entitled *Mathematics: Loss of Certainty*. His "truth" is purely cultural, having been agreed to by "fiat" or by a social contract. Again, in the impressive work of Philip Kitcher entitled *The Nature of Mathematical Truth* there is an argument against the very notion that mathematics is a priori knowledge at all. Instead, it proposes an account of mathematical knowledge which is empiricist at its foundations and which accounts for present day mathematics knowledge in evolutionary terms. Another example is the general agreement among the authors in the anthology *New Directions in the Philosophy of Mathematics* that, since the attempts to found mathematics have failed, philosophers of mathematics should content themselves with a study of the practice of mathematics and abandon, at least for the present time, any metaphysical or foundational analysis of the discipline. The editor of this anthology calls this "new direction" "quasi-empiricism."

Yet, we are Christians as was Aurelius Augustus and I believe that the great Church Father has some ideas which are worthy of our attention. What should we say by way of analysis about Augustine's views? Does the fact that Augustine's philosophy of mind was influenced by Platonic thought disqualify it from serious consideration by Christians today?

First, I want to make a disclaimer or two about Greek philosophy in general. There is no doubt at all that there is a great gulf between the God of the philosophers and the Triune God of the Scriptures. There is also no doubt that there is a great gulf between the Greek notion of the body as a prison house for the soul and the Biblical emphasis upon the body in the Incarnation and in the resurrection body. Finally, there is a great gulf between the Greek notion that the essence of man is to be found in his reason and that the good life is an ascent to the wholly Other and the Biblical view that man is saved from his sin by the grace of God who comes near to us through Jesus Christ and the good life is a response to this salvation. Thus, any return to mathematical realism through exclusively Greek ideas will be insufficient and unsatisfactory. But what about Augustine's views? Are they a good starting point for a satisfactory defense of mathematical realism from a Christian perspective?

I shall list some aspects of Augustine's position which are the source of the most serious objections to his view.

1. Augustine is very optimistic about the possibility of certain knowledge. Furthermore his view depends upon the notion that God can play an important role in an account of man's knowledge. In this account Augustine refers to the Mind of God and says that the principal ideas are found in the mind of God. But some Christians will say that, as the result of man's sin, they are very skeptical about any assertion that man can find the truth through reason even if God is the alleged source of such capacity. Furthermore, some Christians will say that it is improper for us to use God in an explanatory way since, in essence, God is wholly incomprehensible, and so far above our understanding that even our analogies about him have very little value as such explanations. There is the good question of just what effect the coming of sin in the world had upon man's capacity to know the truth. What effect did the coming of sin have upon the image of God in us?

2. Augustine has a rather high opinion of man and his relation to God's other creatures. His views clearly differentiate mankind from the animals, for example, and place mankind in a unique position in the creation order; a creature of God with a special role to play and whose existence is immortal. Second, Augustine places strong emphasis upon man's reason as a most important characterization of the image of God in us. Some Christians may demur at this point.

3. Of particular concern to many Christians is the dualism which Augustine advocates in his view of man; man has an inner self and an outer self; a soul and a body (in more common theological terms). Furthermore, Augustine gives a prominence to the inner man over against the outer man in his explanation of human knowledge. It is alleged, at least, that the Bible does not clearly describe man in such terms, but rather, has a more holistic view of man and that the Biblical notion rooted in Hebraic terms is be, more in terms of the 'heart' rather than the hierarchical view of Augustine.

It was of some interest to me to look into the writings of John Calvin to see if there is any such dualism of body and soul in his writings. In Book I and Chapter XV there is a lengthy discussion about the state of man at his creation: the faculties of the soul, the Divine Image, free-will and the original purity of his nature. In fact, Calvin also refers to the "pagan philosophers" and even grudgingly admits that the sentiment of Plato is "more correct, because he considers the image of God as being in the soul." I shall quote from Chapter XV in section II.

"That man consists of soul and body, ought not to be controverted. By the 'soul' I understand an immortal, yet created essence, which is the nobler part of him. Sometimes it is called a 'spirit'; for though, when these names are connected, they have a different signification, yet when 'spirit' is used separately, it means the same as 'soul'; as when Solomon, speaking of death says that 'the spirit shall return unto God, who gave it'."

Later when referring to Stephen he asserts that the Bible intends "no other than that, when the soul is liberated from the prison of the flesh, God is its perpetual keeper. Those who imagine that the soul is called a spirit because it is a breath or faculty divinely

infused in the body but destitute of any essence are proved to be in gross error by the thing itself and by the whole tenor of scripture. It is true, indeed, that while men are immoderately attached to the earth they become alienated from the Father of lights, are immersed in darkness so that they consider not that they shall survive after death, yet, in the meantime, the light is not so entirely extinguished by the darkness but that they are affected with some sense of their immortality.”

Later he goes on to say the following. “But the agility of the human mind, looking through heaven and earth, and the secrets of nature and comprehending in its intellect and memory all ages, digesting everything in proper order, and concluding future events from those which are past, clearly demonstrates that there is concealed within man something distinct from the body ... We apprehend what is right, just, and honest, which is concealed from the corporeal senses.

Further on in the section he says that “it would be folly to seek for a definition of the soul from the heathen philosophers, of whom Plato is almost the only one who has plainly asserted it to be an immortal substance ... The sentiment of Plato, therefore is more correct, because he considers the image of God as being in the soul.”

It is clear to me that there is within the Reformed tradition the view of some that each person has a body and a soul (spirit) with an essence that is distinct from the body. At least some in our tradition, including Calvin, assert that the Bible teaches that the image of God has its locus in the soul and that this soul is immortal and it is what continues to exist after our earthly death. Such a theological view of man can be the foundation of a Christian epistemology which may include some aspects of mathematical realism.

In conclusion, let me raise some questions for our consideration.

Is Augustine’s view so infused with Platonic notions so as to be unacceptable to Christians today? Does it seem plausible that we Christians will need to develop a philosophy of mathematics which will be totally unacceptable to the existing mathematical community because it uses the Triune God of the Scriptures in an essential way in its explanations? What should our attitude be to the current views about the nature of mathematics which limit the categories of explanation to human culture and social evolution. Will these views lead us into deep skepticism about human knowledge and into profound relativity or are they more compatible with a Christian position than Mathematical Realism?

Is the Christian Mathematical Realism as originated by the Bishop of Hippo a viable option for a Christian philosophy of mathematics today?

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