Review of Naming Infinity: A True Story of Religious Mysticism and Mathematical Creativity Paul J. Zwier, Calvin College

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NAMING INFINITY: A True Story of Religious Mysticism and Mathematical Creativity, by Loren Graham and Jean-Michel Kantor, The Belnap Press of Harvard University Press, 2009, 227 pages, notes, hardcover, ISBN 978-0-674-03293-4, \$25.95.

Loren Graham is a specialist in the history of Russian science who has written many books and articles on the subject. One such book *Science and Philosophy in the Soviet Union* was a finalist for a National Book Award. One of his most recent books is *Russian Religious Mystics and French Rationalists; 1900-1930.* Jean-Michel Kantor is a French mathematician whose main interest is topology and is a popular writer on science. His website details his interest in the *diffusion* of science. (I will use G-K for the book's authors.)

Prelude

As students of mathematics we invariably confront infinite sets. We learn about the natural numbers 1, 2, 3, 4, ..., but, early on, we form the infinite set, $\{1, 2, 3, 4, ...\}$ where we complete the formation of natural numbers in *our minds* and append the ellipsis with the symbol $\}$ to form the set, N, of **all natural numbers**. Continuing this process, we study and name the following infinite sets: the set Z of integers, the set Q of rational numbers, and in an explosive burst- to include the set A of all algebraic numbers, and the set of all transcendental numbers, T, finally arriving at the set of real numbers, R, so familiar to scientists today.

In a similar way we form the set of all US States and we say that the cardinal number of this set is 50. But, since mathematics is the science of the infinite, we dare to take the position that every set should have a cardinal number and that sets have the same cardinal number if and only if they can be put into one-to one correspondence.

In 1873, the German mathematician, Georg Cantor published a paper in the *Crelle Journal* which proved that the set *R* of the *continuum* of real numbers is non-denumerable; that is, there is no one-to-one correspondence from the set *N* to the set *R*. Furthermore, he proved that there is a one-one correspondence between *R* and the set of all subsets of *N*. Cantor named the cardinality of the natural numbers \aleph_0 and the cardinality of *R* by the German letter *c* and later also by 2^{\aleph_0} .

Later, in his now famous speech given to the International Conference of Mathematicians at Paris in 1900, David Hilbert posed as his first problem (of 23) whether

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there are any non-denumerable sets whose cardinal numbers lie between \aleph_0 and *c*. He proposed the name \aleph_1 , for the first such, the name, \aleph_2 for the second such and so on. The Continuum Hypothesis is that $\aleph_1 = c$.

As is obvious from the following quotation, this book breaks some new ground in the way that this history of mathematics is written.

This book is devoted to a little known but exemplary episode in the recent history of the relationship of mathematics and religion, all within the context of much larger issues of religious heresy, rational thought, politics, and science. It is intended for general readers, although we hope that mathematicians will also find it worthwhile. It is the story of an initial breakthrough by a German mathematician (Georg Cantor), that was picked up and developed further by the French, who eventually stalled, but who taught the new development to Russian mathematicians; the Russians then returned to their homeland to push onward to a fundamental insight.

At the center of the story is an encounter at the beginning of the twentieth century between mathematicians on set theory and the religious practices of the heretical Name Worshippers in Russia. Set Theory was, at first, developed in France but then underwent a profound crisis, only to have the Russians enter the scene with new energy. We will describe how two different states of mind connected with two different cultural contexts led to contrasting results; French skepticism and hesitation, Russian creativity and advancement. A central idea of this book is that religious heresy was instrumental in helping the birth of a new field of modern mathematics.

I suggest that the book is **multi-dimensional** in its treatment of the various topics it considers. I shall discuss a few of its dimensions.

• A comprehensive look at the personalities There are deep and detailed biographies of some of the mathematicians featured, which includes their family history, their education, their personalities, their mathematical work, their foibles (including sexual preferences and practices), their illnesses, their psychological struggles, and the untimely deaths of some of them.

• The Set-Like Structure of the Book The first set of importance is a singleton consisting of the German, Georg Cantor, the second set is a singleton consisting of the German, David Hilbert. The third set is a trio consisting of the French mathematicians Rene Baire, Emile Borel, and Henri Lebesque while the fourth set is a trio of Russian mathematicians consisting of Pavel Florensky, Fedor Egorov, and Nicholi Luzin. There is another set of 661 monks who stated that they did not support the doctrine of "Name Worship", and another set of 517 monks living in the same monastery who supported the doctrine and also declared that they would remain there till death.

• **Pictures and Illustrations** The book features a gallery of some 36 illustrations which are scattered in the commentary. Yes, the gallery includes formal pictures of the

mathematicians who played important roles in the story including an unflattering picture of the villain in it. But there are several others which will be of interest to the reader.

There is a photograph of the St. Pantaleimon monastery on Mount Athos in Greece, and also of one of the buildings of the Moscow State University where the Mathematics Seminars were held. There is a picture of Egorov's gravestone in Kazan, the city where he was exiled. and also a sketch of the Genealogical chart of the Moscow School of Mathematics.

For first time readers, it may be helpful to view this gallery of photographs as one begins to read the book. Each photograph plays an important role in this gripping story. I found that I returned to the gallery again and again since it contributed to make the story come alive.

• Worship and Prayer An important entity which plays a pivotable role in this history is the famous Jesus prayer

Lord Jesus Christ, Son of God, have mercy on me, a sinner.

As practiced in the Eastern Orthodox tradition it is intended as a way to obtain quietness and peace by physical and mental fusion with God by combining hundreds of repetitions of short sequences of the same words. There are three stages of immersion in praying this prayer. First the words are intensely heard by the worshipper, but then the prayer enters the mind of the believer, with the result the mind clings to the words so that the worshippers find themselves in the presence of God, and finally the prayer goes to the heart of the worshipper giving illumination with the result that the person achieves a oneness with God.

• Heresy and Controversy Now comes the controversial part. Does the name Jesus Son of God become identified with God through this fervent worship? The Eastern Orthodox Church has always said "No!" to this question and has declared that this view of the Name Worshippers is heretical. The Name Worshippers, including the theologian-mathematician, Pavel Florensky and Dmitri Egorov believed "Yes".

• **Historical Contrast** The authors add the most important dimension to the story by describing the historical event that the French trio in their choices and practice of mathematical work proceeded in another direction from that of the Russian trio. The result was that the French did not continue to contribute to the deciding of the Continuum Hypothesis while the Russians became enthusiastic participants in such research.

• **Philosophical Explanation** Now comes the interesting part of the book.

The reason given by G-K that the French trio changed the direction of their research is that they began to see that the problem posed by Hilbert was very hard and required new techniques in defining uncountable subsets of real numbers. True, they had decided to use the context of Axiomatic Set Theory, ZF, as developed by Zermelo and Fraenkel for their work. After becoming aware of some hidden assumptions they had made in their arguments and in hearing about some of some possible paradoxes in Axiomatic Set Theory, they lost their verve and nerve for the problem, and expressed such publicly. Graham and Kantor attribute their

judgment to their **rationalism** as developed by Rene Descartes and also to the philosophy of Auguste Comte known as **positivism**. Thereafter, they discontinued their work on the problem.

The Russian trio consisted of two Name Worshippers: Florensky, the theolgianmathematician and the mathematician, Egorov, and a third member, Luzin, who had often traveled to France and was aware of the work of their prominent mathematicians. G-K document the fact that Luzin was at a low point in his life. He had lost his zest for mathematical research. Notwithstanding, he read the theology of Florensky as found in his now famous work *The Pillar and Ground of the Truth* and the manuscript for *Holy Renaming*. Along with this, his letters show that he read Plotinus, and William James. The result was that he became a Name Worshipper! Because of his conversion he discerned the value of *naming* certain uncountable subsets of real numbers and proving theorems about them. By doing so he created the area of mathematical research called Descriptive Set Theory. G-K summarize their historical findings in the following quotation (p. 192).

The Russians who developed descriptive set theory and assigned names to subsets of the continuum posed the possibilities of the existence of new entities in the mathematical universe, and they went on to provide a program for future research which resulted in substantial agreement of mathematicians all over the world about the new entities. That achievement might have occurred without the inspiration of a religious heresy, but as researchers loyal to the historical record, we maintain that the way it actually happened was within a context of mystical, Name Worshipping stimulation.

This book will take mathematicians and interested scientists on a fast-paced, intriguing, challenging but enjoyable journey. Graham and Kantor have indeed told a true mathematical story with a well-documented interpretation, a Russian view of the infinite in mathematics. I predict that readers of this book from the ACMS community will find it a terrific read. Furthermore, I believe that some scholars in this Christian community might want to want to discuss, analyze, criticize, or amplify the argument of this well-written book. Theologians who read the theological essays of Florensky will better understand some fundamental doctrines and practices of the Eastern Orthodox Church of the early 20th century which will benefit us today.

I will give Pavel Florensky the final word. What follows is a quotation from his book, *The Pillar and Ground of the Truth*. Maybe this theological statement is what the Russian mathematician Nicholi Luzin needed to read!

Neither "the contradictions of the Holy Scriptures and the dogmas" nor "spiritual illuminations" contain anything absurd and therefore if both an honest rationalist and an honest mystic refer to them they do in fact exist. But that which is a contradiction, and an unquestioned contradiction, for the ratio, stops being a contradiction at the highest level, is not perceived as a contradiction, is synthesized. And then, in a state of spiritual illumination; there are no contradictions. Therefore, there is no need to try to convince a rationalist that there no contradictions: they exist, they are unquestionable. But a rationalist must believe a mystic when the latter states that the contradictions turn out to be a higher unity in the light of the Sun that does not set, and then they precisely show that the Holy Scriptures and the dogmas are higher than fleshly rationality, and thus could not be thought up by man; i.e. are Divine (p 358).

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