# Motivating students in their study of Mathematics

Johan H de Klerk
Mathematics & Applied Mathematics
North West University (Potchefstroom Campus)
Potchefstroom, South Africa
johan.deklerk@nwu.ac.za

## 1. The role of a contextual approach in teaching mathematical subjects

Some people would argue that there are certain subject fields – for example pure Mathematics – that could be viewed as separate entities divorced from all else in reality. Other people would argue the opposite, saying that a subject field cannot, and does not, stand alone.

The first viewpoint becomes clear by consulting the majority of (especially) older text books on Mathematics, in which a very strong formalised form of presenting mathematics is normally used. The second viewpoint is perhaps the one that is favoured by most applied mathematicians. As lecturer of Applied Mathematics, I also adhere to this second view.

In my view, a subject field, and especially a natural science subject, can be viewed in a setting of expanding contextual frames. The more technical subject matter may then be viewed at the centre, with contextual frames of history, natural science, technology, society, nature and religion surrounding it. In my classes, I make ample use of this pedagogical instrument to bring more perspective to the mathematical subject matter itself.

The background to this approach is the question of whether a subject – Mathematics in this case – can be taught from a Christian perspective. My answer to this question is that it can be done (De Klerk, 2004). For that, my point of departure is that a subject should be viewed in context with other, wider aspects. Let us call this view the *view of science in context*. Stoker (Stoker, 1976:135) comments as follows on this viewpoint: "The above provides us … with an argument for the necessity of fulfilling our Christian calling of doing science, we being committed to do science within the context of … our Christian life and world view – and in accordance with the relational demands of the various fields of research of the various sciences."

In the present discussion, however, attention will not in the first place be drawn to such a contextual approach. What is of importance now is the context of the history of mathematics and, as an extension, also the history of technology. This context will be used as background and pedagogical tool in motivating students in their study of the subjects Mathematics and Applied Mathematics. This remark brings us to the formulation of the main thesis of this discussion.

The thesis of this article is as follows: It is to a student's benefit if the pedagogical tool of the history of mathematics and technology is used to motivate students in their study of Mathematics and Applied Mathematics. This thesis will be argued and it will be backed up by discussing results of an end-of-term class evaluation in which the views and feelings of students were tested.

This article will be structured as follows: In section 2 the role of the history of mathematics and technology in class discussions will be discussed. Terminology and viewpoints from the field of technology will form the introduction to section 3. In the rest of section 3, attention will be paid to examples of assignments illustrating what insights may be gained by relating aspects from the history of mathematics and technology to the subject itself. In section 4, feelings and views of students on these assignments will be given and discussed. Finally, in section 5, the article will be concluded with a summary and a view to future class strategies (and the use of other contexts).

## 2. The role of the history of mathematics and technology in class discussions

It is widely argued (De Klerk, 2004) that it is to the benefit of students if aspects of the history of an academic subject can be integrated as a tool in teaching the subject (see also, for example, Furinghetti *et al.*, 2004; Hairer & Wanner, 1997; Katz, 1993; Kauffman, 1991; Serres, 1995). Some of the general arguments that are usually given to motivate such a choice are the following: It shows the "human" side of Mathematics, it softens Mathematics, it motivates students in their study of Mathematics, etc.

The inclusion of aspects of the history of a subject in the course material itself might perhaps raise certain questions. The main question is of course: does a subject stand separately from its history, or are the two integrated? It seems that there are, internationally speaking, different answers to this question. With respect to the subject Mathematics, Van Maanen (Van Maanen, 1999) makes the following comment: "In some parts of the world a different relationship between history and mathematics may have been developed. For example, in Denmark and Sweden, history of mathematics is regarded as an intrinsic part of the subject itself." For our discussion, we will assume that at least some aspects of the history of the subject and the course material itself cannot be separated totally from one another. In my own courses, I find it fruitful to include some historical aspects.

For more than a decade now, I have embarked in my classes on a road of discussing and integrating aspects of the above-mentioned contexts. Of course, this also includes the history of the subject – and this has been done with a certain degree of success.

Unfortunately, a negative aspect of this approach should be mentioned here: At least in my country, South Africa, young people generally tend to be very unenthusiastic about History as a school subject. As one can expect, this matter is also reflected in the interest of university students when it comes to the history of science (and in the present case, mathematics). On the one hand, this might perhaps only be a local phenomenon. On the other hand, in the light of the following remark by Kauffman, it seems as if it is rather a more general, international situation. Kauffman (Kauffman, 1991:185) points out with respect to his subject field, Chemistry: “In short, most students of Chemistry, in common with their instructors, have only minimal interest in, or knowledge of, the history of Chemistry.” This can certainly be generalised to include Mathematics.

I realised therefore, that for class purposes, one has to take a wider look and try to find an additional pedagogical tool to counter these negative views. At the end, I found such a tool in the following approach. Since the majority of students in our Mathematics and Applied Mathematics courses are students in engineering, computer science and financial mathematics, many of them do not have much motivation for studying Mathematics (and even less so for its history). However, these students usually have a keen interest in many technological matters, even in the history thereof – for example in such topics as the “race” to put a man on the moon, the development of computers and the building of super-suspension bridges. Therefore, some additional aspects were introduced, specifically concerning the history of technology. In this manner it was found that students became much more motivated for, and interested in, their mathematical studies.

Some results of the approach of discussing and integrating aspects of the history of mathematics and technology in the Mathematics course should be mentioned here.

One result is that brilliant students tend to become still more interested in the course material. One student, for example, brought me a DVD recording of television programmes on interesting technological matters. On one of these programmes, the idea is discussed of building a bridge over the Bering Strait between Alaska and Russia.

Still another result is the following: Over time it was found that a discussion of technological subjects inevitably leads to a discussion of deeper technological-ethical and technological-societal concerns. Such discussions are of course precisely the type of thing that one would hope for in teaching at university level and are in line with general educational views.

Conway and Riggs, for example, in a discussion on valuing in technology, start with the following general and often-heard viewpoints: space technology is a symbol of man’s ability; far more time and money should be invested in it; and this type of technology pushes back the frontiers of our knowledge and control. They than argue that instead of proceeding along these lines, a deeper pedagogical question should be (Conway & Riggs, 1994:227): “What are your reactions to this statement?” For them the emphasis is therefore not on the *technological achievements* as such, but on the *views* on the technological achievements. Technological information therefore fades, while ethical, moral and even religious views relating to technology become more important.

It is therefore clear that when one starts to talk about the history of technology, at the end one cannot avoid discussing topics from the philosophical side of technology as well. This theme will form the introduction to the next section.

**3. Circumstances in class**

In the light of the above remarks on the ethical and moral side of technology, and with a view on class circumstances, it is necessary to present a short discussion on some topics relating to the philosophy of technology. This is also done in class to give the necessary background. Of course, the idea is not to enter in a deep philosophical discussion of technology. The idea is rather to emphasise the fact that in discussing the *historical* side of technology in class, one cannot escape from the *ethical*, *societal* and even *religious* sides of technology.

In class discussions, the concept "history of technology" is viewed broadly. It is important that students should realise that the history of technology entails much more than a historical description of techniques, and that there is also a philosophical side to technology. In this respect, the work of the engineer-philosopher Egbert Schuurman proves invaluable. One of his main viewpoints concerning the development of technology responsibly is formulated as follows (Schuurman, 1995:6): "… we must know the *history of technology*, and particularly of the *spiritual forces* that drive technology" (italics added). This is also a good starting point for a class discussion.

The history of technology is divided by Schuurman (Schuurman, 1995:6) into three main eras, namely:

- the *traditional era* (that lasted until about 1500 AD),
- the *practical era* (from about 1500 to 1800) and
- the *modern era* (from about 1800 to the present).

Schuurman calls an overestimation and overemphasis of technology *technicism* – this term is also used in class. Some examples of such processes of making everything technological, quoted by Schuurman are the following (Schuurman, 1995:86): the dehumanisation of labour, the devastation of nature, the pollution of the environment, problems relating to farming and meat production, and problems relating to the genetic engineering of humans. Of course, these topics speak clearly for themselves, and it is of importance if one can get students to start thinking about such matters.

The field of technicism is vast, and the following are examples of danger signs:

- the view that "technologise" is the *answer* to all questions,
- the idea that "technologically possible" is the *light* for the future, and
- the viewpoint that man is the *servant* of science and technology.

In section 4 we will come back to the last viewpoint.

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Before discussing the practical class situation, one other question also needs to be addressed. This concerns the logical place of aspects of the history of mathematics and technology in a class discussion. This kind of question is usually raised by people who do not regard it as necessary to discuss Mathematics in its broader setting.

One of the characteristics of Applied Mathematics – apart from its mathematical character – is its basic problem-solving character. This pillar, which is of course also emphasized in teaching the subject, is called *mathematical modelling*. It concerns the basic structure of the subject Applied Mathematics; for practical purposes it can be divided into the following three stages:

- Stage 1: the *reformulation* of the physical (or real world) problem as a mathematical problem.
- Stage 2: the *solving* of the mathematical problem.
- Stage 3: the *evaluation* of the mathematical results.

In the first two stages, attention is usually paid to the more formal, mathematical side of the problem. The first stage usually takes place via a process of mathematisation of a given problem from either nature or culture (for example planetary motion or deflection of beams). The real application of mathematical theories and other techniques takes place in the second stage. Both stages have specific scopes: *problem formulation* in the first stage and *problem solving* in the second.

The third stage, however, has a wider scope, namely evaluation. It usually offers one an opportunity for broader discussions than the first two. Except for interpreting the mathematical results in this stage, the correlation between the mathematical results and the physical problem from nature or culture also has to be evaluated. This stage of the mathematical modelling process also gives one the opportunity of discussing further aspects of the physical problem in its wider historical, scientific, societal, ethical and religious contexts. As an example, the following can be cited from one of my study guides, specifically relating to the historical context (De Klerk, 2007):

"Most of the mathematicians that helped to develop the theory of differential equations lived in the period now referred to as modern rationalism. Their work clearly shows the trends of their time, namely the mathematisation of the natural sciences. Faith in this involved faith in the rational abilities of humankind; and the influence of the church was shifted more and more into the background."

The pedagogical strategy of emphasising the history of the subject has been implemented in a specific class group. I would like to mention the following points with respect to this course:

- <u>Degree</u>: The course forms part of a series of numerical analysis courses in the curriculum of a general BSc degree in the mathematical sciences. The course is also prescribed for engineering students – as such it also acts as a service course.
- <u>Course title</u>: "Numerical solution of partial differential equations".
- <u>Class information</u>: The course is a third-year Applied Mathematics course with an attendance of about 200 students, average age about 20 years. It is presented annually during the first semester, running from the beginning of February to the end of May.
- <u>Course material</u>: About three quarters of the course material covers the numerical solution of the "big three" second order partial differential equation problems, namely

- parabolic equations (with the *heat equation* as protoproblem),
- elliptic equations (with the *potential equation* as protoproblem), and
- hyperbolic equations (with the *wave equation* as protoproblem).

The historical root of differential equations dates back to the 18th century when the mathematician Leonhard Euler first mentioned such equations in his works, and when there was the first success in solving the so-called “vibrating string problem” (the above mentioned wave equation).

The expressions *heat equation*, *potential equation* and *wave equation* already hint to the fact that these terminologies are used in some physical, and therefore historical, sense. A discussion of the history of these mathematical problems, together with their solutions, immediately brings students to the realisation that Mathematics as subject also has its developments, its branches and its retreats. In this way the close relationship between history and subject matter can be emphasised. This view is very different from the one that students generally come to adopt after studying a handbook on a mathematical subject where the Mathematics is nicely formalised, coherently systematised and logically presented. Furthermore, such a discussion also lets students understand the close relationship between the subject matter and the history of the subjects.

In the rest of this section, some specific examples (in the form of class assignments) will be given and discussed in order to illustrate the pedagogical strategies that are used in teaching this course. I developed these assignments over time and I found that they provide good opportunities for class and private discussions. The assignments are given on a regular schedule during the course. One should remember that the main objective of any course in Mathematics or Applied Mathematics is the mathematical content itself. The main objectives of these assignments are to motivate students and to give them the opportunity of finding pleasure in each task. Therefore one cannot expect a high level of research from these assignments. The moment one expects too much in this respect, it becomes another burden on top of the normal mathematics homework, and then one totally misses one’s aim.

Each of the following assignments is presented in the format “background”, “learning objective” and “result”. With these examples, this paragraph is closed. In the next paragraph this pedagogical tool will be evaluated and discussed.

<u>Assignment 1</u>

<u>Background</u>: In this introductory assignment, every student is asked to discuss any interesting technological development from any of the traditional, practical or modern eras of technology.

<u>Learning objective</u>: The learning objectives of this open-ended assignment are to give students the opportunity:

(a) to identify and describe at least one of their own technological interests, and

(b) to personally do some reading on the history of such choices.

Remark: In light of the remarks made earlier, no high level answer is expected here, and therefore, no guidelines are provided. The objective at its deepest level is: tell what interests you! For me, as lecturer, it has given, and continues to give much insight into the minds of young people and it has also opened up a whole new world.
Result: Due to the large number of technological tools of all eras, but especially of the modern era, the group of students reported on a wide variety of interesting topics. Some examples are as follows. From civil engineering: the Milau Bridge in France; from electronic engineering: holography and mobile phone technology; from mechanical engineering: sun panels; from mechanics: tandem bicycles; from planetary motion: space robots. Most of the examples are from the modern era of technology, of course, as one could have expected.

## Assignment 2

Background: In the subsequent task every student again has to discuss an interesting technological development, but this time with the restriction that it has to be a development from the traditional era, and in particular before 1300.
Learning objective: By excluding all modern technological developments, the learning objectives of this assignment are:
(a) to go further back in history to earlier technologies,
(b) to let the history of technology show some societal implications,
(c) to discuss the role that technology played in earlier and modern societies, and
(d) to compare the role that technology played in earlier years to that of today.
Remark: In both old and new technological products, attention can of course be drawn to the importance of the societal side of technology. As an example one may consider the development of stirrups. The importance of a stirrup lies of course in the revolution it brought about in warfare. In this respect, Gordon (as quoted in White, 1962:1 et seq.) remarks: "… without stirrups, your slashing horseman, taking a good broad-handed swipe at his foe, had only to miss to find himself on the ground". Similarly, a spear can be thrown much farther by a horseman using stirrups than otherwise.
Result: Due to the era restriction, all technologies of the practical and modern era are ruled out. Also in this case a large number of interesting technological objects were reported on, for example, aqueducts, calendars, cranes, farming techniques, Greek fire, music instruments, paper-making, pinhole cameras and stirrups.

## Assignment 3

Background: In this assignment the focus returns to technologies of the modern era. The task now is a discussion of super structures, particularly the tallest building in the world. (At the start of the last presentation of this course, the Taipei 101 Building in Taiwan (Taipei 101, 2006) was still the tallest building; however, on 21 July 2007 the Burj Dubai building in Dubai became the tallest building in the world ( Burj Dubai, 2007).)
Learning objective: The learning objectives of this assignment are:
(a) to make students aware of the architecture of the building,
(b) to discuss some of the mathematical topics relating to this building, and
(c) to emphasise the historical, societal and moral aspects of such structures.
Remark: Relating to these learning objectives, the following notes could be made pertaining to the way the different goals are achieved:

(a) *architecture*: the design of the building was inspired (i) by traditional Chinese pagoda architecture and (ii) by the form of a giant shoot of the bamboo plant;
(b) *mathematical topics*: the building has (i) the two fastest lifts in the world, reaching a speed of 63 km/h and (ii) an 800 ton tuned mass damper at the 88$^{th}$ floor that stabilises the tower against earthquakes and typhoons;
(c) *historical, societal and moral aspects*: this goal can be achieved by discussing the general view that high towers are not only built to be landmarks and tourist attractions, but that they are also constructed to be great monuments of our time; or, as it is explicitly formulated by Dupré (Dupré, 2001:8): "… towers are for power".

Result: Due to the importance of this theme – regarding the construction process as well as the idea of monuments and power – opportunities arise for class and private discussions of ethical, moral and philosophical viewpoints.

Assignment 4

Background: In the next task, a study has to be made of the development of musical instruments (especially string and percussion instruments) and the development of the mathematical model describing the physical problem.

Learning objective: The learning objectives of this assignment are:
(a) to relate the history of the mathematical and technological matters directly to the wave equation course material; and
(b) to pay attention to the relationship between Mathematics and the broad fields of culture and nature.

Results:
(a) Due to the importance of the close relationship between especially Applied Mathematics and the physical world (the mathematical modelling process), this task offers a good opportunity to emphasise this relationship.
(b) In his book "Mathematics: Is God silent?", Nickel (Nickel, 2001) writes about the silence of God's voice in Mathematics. One of his recommendations (Nickel, 2003:5) to counter this situation, is to "tie the subject to the physical creation …". The present pedagogical strategy is exactly in line with Nickel's remark.

## 4. Evaluation and discussion of the pedagogical tool

At the end of the course, it is necessary to evaluate students' feelings and views on the given assignments and discussions. One last assignment, in the form of an evaluation, is therefore given to assess the pedagogical tool.

Evaluation

Background: To evaluate the views and general feelings of students on these assignments, the following questions have to be answered:
(a) The designer and builder of the Suez Canal, Ferdinand De Lesseps, remarked as follows (as quoted by Cadbury, 2003:198): "… science has declared that the canal is possible and I am the servant of science". Do you agree with De Lesseps' sentiment of being a servant of science?
(b) Is it possible to overemphasise technology? If so, give specific examples.

(c) Did the different assignments on the history of mathematics and technology help you to develop a broader view on life and the world in general?

Purpose: The purpose of this task is :

(a) to ascertain the feeling and views of students with respect to the pedagogical strategies used in class, and

(b) to measure these views on three levels, namely:

( i) the *practical technological* level,

( ii) the overestimation, or *technicism*, level and

(iii) the general *worldview* level.

Result: The answers are summarised in Table 1.

| **Question** | **Yes** | **Yes/no** | **No** |
|---|---|---|---|
| 1. Do you agree with De Lesseps' view? | 44% | 15% | 41% |
| 2. Is it possible to overemphasise technology? | 88% | 6% | 6% |
| 3. Did these tasks help you to develop a broader view? | 90% | 5% | 5% |

Table 1: Results of students' views

In all three cases the number of respondents was between 130 and 140, and therefore large enough to produce significant results. All answers are interesting, but the reaction to the last question is in particular overwhelmingly positive. The following specific comments can be made:

Question 1 (the practical technological level): In this question De Lesseps' use of the word "science" should probably be understood in the sense of "technology". The reaction to this question shows the typical view among young people studying engineering, computer science, etc., namely that science/technology is "master" and "I am its servant". As far as possible care was taken not to influence students in their answer to this question.

Question 2 (the overestimation level): With respect to the high percentage (88%) of students that have the view that it is possible to overemphasise technology, the following remarks can be made: On the one hand, it might have happened that there was a degree of influence by previously mentioning examples like the dehumanisation of labour, the devastation of nature and the pollution of the environment. On the other hand, young people are usually well aware of problems created by the overuse of technology, for example in the form of problems relating to global warming. Therefore, an answer like this could have been expected.

Question 3 (the worldview level): The positive reaction to the question relating to the development of a broader view on life and the world may serve as a motivation to continue with this pedagogical strategy. The following two remarks can be made: First, there is unfortunately no information to which this result could be compared, as a similar question was not asked at the beginning of the course as well. Second, although this question is formulated with respect to a "broader view", the positive reaction towards life in general can, possibly, also be interpreted as a positive view towards the mathematical class situation. Students in this class, whether science or engineering students, also take a course in Philosophy of Science – such a response is therefore not totally strange.

**5. Summary and conclusion**

The low level of motivation among students towards their studies of Mathematics and/or Applied Mathematics, and how to counter it, forms the back bone of the present discussion. One especially notices such motivational problems among students that are “forced” to study a specific subject outside their main course.

Integrating aspects of the history of the subject and of technology with the mathematical subject matter itself, it became clear to me that students are more positive with respect to their mathematical studies. In section 1, this matter was formulated as the main thesis of this discussion. In the rest of this discussion, this matter was discussed from several sides. With the discussion of the results of the evaluation in section 4, the conclusion was drawn that this pedagogical tool functions well as a motivational tool.

The positive reaction of students to the discussed pedagogical strategy brought one to the point of thinking about further enhancements to this strategy. The following possibilities may be mentioned:

- A first possibility – and it has already been put into practice – is the presentation of a PowerPoint series. One’s general expectation is of course that such presentations would be received positively, especially since it is brought in “a modern technological idiom”.
- A second possibility – which has not yet been acted upon – is to deepen the academic level of these assignments. In a sense, it is contrary to the “rules” that I have set for myself of not putting too much of a burden on the shoulders of the students. Then again, it might interest students to do some more structured reading on technology. Two kinds of texts come to mind: the first with a more philosophical touch, like the work of Schuurman (Schuurman, 1995) and the second with a more practical touch, like the work of Petroski (Petroski, 2004).

Results of these ideas and further extensions to other contexts (among others, the contexts of society, nature and religion) may form the basis for further discussions.

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